

Real Options Valuation of Photovoltaic Investments: A case from Turkey

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Abstract: Investments in renewable energy resources have become inevitable due to increasing energy demand and energy prices, diminishing non-renewable energy resources, and the outgrowth of carbon footprints. Photovoltaic (PV) systems offer high solar energy potential in sustainable energy production whereas their high initial costs necessitate critical strategic valuation of investments. Valuation with conventional methods has been challenging due to existence of uncertainties such as fluctuating PV panel prices, changing meteorological conditions with certain effects on power generation, and governmental policies on energy market regulations. This study aims to propose a real options approach to valuation of residential rooftop PV system investments considering these uncertainties and demonstrate benefits of this approach with an application on the residential PV investment decisions in Turkey. The proposed method, Real Options Valuation (ROV) with Least-Squares Monte Carlo Simulation (LSMC) considers the deferral option of the investor by utilizing stochastic simulations, the discounted cash flow method, linear regression, and backward dynamic programming and thus evaluates the effects of uncertainties on financial attractiveness of residential PV investments. The case study findings proved that ROV with LSMC having a 7-years deferral option supported the investment decision with realizable cost-effective options while “NPV method” resulted in an infeasible investment. Scenario analysis was also conducted to explore policy options that can be used to promote solar energy investments in Turkey. This study has a potential to have practical contributions for investors as well as implications for policy-makers.

Highlights

- Considering strategic value of flexibility for investment time with deferral option

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- Handling uncertainties using stochastic simulations
- ROV method can prevent faulty rejection of investments with classical methods

Keywords: Government incentives; Least-Squares Monte Carlo Simulation (LSMC); Photovoltaic (PV) investments; Real Options Valuation (ROV); Residential buildings; Solar energy.

Word count: 10,541

Nomenclature

Abbreviations

GBM	Geometric Brownian Motion
LSMC	Least-Squares Monte Carlo
MJDM	Merton Jump Diffusion Model
NPV	Net Present Value
PV	Photovoltaic
ROV	Real Options Valuation
ROV-LSMC	Real Options Valuation with Least-Squares Monte Carlo Simulation

Notations/Symbols

$C_{inv}(t)$	Inverter cost at any time t
$C_{oth}(t)$	Other investment costs at any time t
$C_{O\&M}(t)$	The yearly operation and maintenance cost
$C_{pan}(t)$	PV panel cost at any time t
$E_c(t)$	The energy consumed by the house between the instant $t - 1$ and t
$E_g(t)$	The energy generated by the PV system
$E(NPV_{flx})$	The expected value of the NPV_{flx} values generated from different paths
$E(NPV_{trd})$	The expected value of the NPV_{trd} values generated from different paths
$F(T, w)$	The value of the call option at time T along path w
i	The discount rate determined by the opportunity cost of capital
$I(t)$	The initial investment cost at time t
$NPV_{flx}(w)$	Net Present Value, including the option value
$NPV_{trd}(w)$	Traditional Net Present Value
$O_{O\&M,r.rate}$	The yearly reduction rate for operation and maintenance cost
$O_{r.rate}$	The reduction rate for other costs
$P_{bill,w.PV}(t)$	Electricity bill after the PV system installation at the instant t
$P_{bill,w.o.PV}(t)$	Electricity bill before the PV system installation at the instant t
$P_{dst}(t)$	The cost of electricity transmission from the power plant to the house at the instant t

$P_{et\&d}(t)$	The electricity tariff and distribution price at any time t
$P_{trf}(t)$	The tariff price of electricity at the instant t
r	The risk-free discount rate
$R(t)$	Revenue at the instant t
$R(t) - C_{O\&M}(t)$	The revenue function for any time step between the initial investment time and the service life of the investment
T	The option maturity
T_{life}	The service life of the investment
w	The number of generated paths
W_t	Wiener process with a mean equal to zero and variance equal to one
α	The growth rate
$\Upsilon(t, T)$	The set of optimal times to exercise the option during the defined time frame $[t, T]$
δ	The standard deviation of the Poisson jump size
λ	The mean number of arrivals per unit time
μ_j	The expected value of the Poisson jump size
σ	The volatility or the standard deviation of growth rate
$\sigma(NPV_{trd})$	The standard deviation of the NPV_{trd} values generated from different paths
$\phi(t_i, w)$	The conditional expectation function
$\ \cdot\ $	The norm of Hilbert vector space from which the estimated value of the continuation function results
$\mathbb{E}_Q[\cdot]$	The risk-neutral expected value operator
$\sum_{i=1}^{N_t}(V_i)$	The compound Poisson process
Units	
kWh	kilowatt-hours
m ²	square meter
MW	megawatt
USD	United States Dollar
kWp	kilowatt peak
TL	Turkish Lira

1. Introduction

Solar energy is a clean, sustainable, and renewable energy source that is generated by harnessing the sun's power through a range of technologies such as photovoltaic (PV) panels, solar thermal collectors, and concentrating solar power systems. These technologies enable the conversion of sunlight into electricity or heat, making solar energy a versatile solution for various applications. One of the promising applications of solar energy is its use in residential settings. The rooftops of residential buildings provide an ideal location for the installation of solar panels, enabling homeowners to generate their own electricity and meet

their energy demands sustainably. This decentralized form of energy generation allows residential solar systems to be directly connected to the grid, offering several advantages over commercial solar production plants. Production at the point of use further reduces transmission losses and allows for greater energy independence. This advantage not only mitigates the necessity for additional electricity transmission networks, but also enhances the grid voltages at the respective location. Another target in power production using renewable energy resources is enabling consumers to meet for their own energy demands and residential PV investments can serve for this aim [1]. Solar energy is also advantageous as being a domestic energy source; it can help to lessen the dependency on foreign energy sources.

Given its high levels of solar irradiance and favorable climate conditions, Turkey exhibits significant potential for the development of solar energy. The country receives an average of around 1800 to 2200 kilowatt-hours per square meter per year, which is among the highest in Europe. The yearly average solar radiation is 3.6kWh/m²-day and the total annual radiation period is approximately 2640 hours [2], which leads to a possible solar energy generation of 380 billion kWh/year [3]. Even though Turkey can be deemed as an ideal location for solar power generation, due to economic considerations it may be difficult to fully establish the required systems to harness its full potential.

Solar energy investments in Turkey began in 2014 with 40MW installed capacity, and by June 2022 the total installed capacity has become 8.479 MW (approximately 8.35% of the total installed power/energy production of Turkey) [4]. In 2020, Turkey was the 16th country in the global installed solar power capacity ranking [5]. Even though renewable energy resources drive almost 54% of the domestic energy production in Turkey, Turkey can only cover 31% of its total primary energy supply and is still highly dependent on energy imports from foreign countries [6]. Therefore, the installed solar power capacity still needs to be increased to reach safe limits in environmental targets [7].

According to Electricity Distribution Sector Report for 2021 generated by Turkish Electricity Distribution Corporation, the industrial areas consume the highest electricity with 45.7%, followed by commercial and residential areas with 24.8% and 23.1%, respectively [8]. Even though the uptake of residential areas in energy consumption is relatively small with respect to other areas, they offer vast amounts of suitable roof areas for PV panel installations due to their architectural nature. By using the rooftop opportunity of residential areas, the installed solar power can significantly be increased. Thus, residential rooftop PV systems are considerable in replacing fossil fuels to a certain extent.

Regardless of the sectoral areas that the PV systems are installed, the investment has still been capital-intensive. When calculating the financial returns to make decision, it is essential to consider the main generated value, which includes the environmental benefits, particularly the reduction in carbon footprint. Although the environmental benefits can be estimated numerically by relating them to carbon prices, it is important to note that these benefits are not considered as tangible income for Turkey in this study. Instead, the savings from the electricity bill, which constitutes a significant portion of the financial returns, has become the main value considered in the financial calculations. The required plant installations to convert solar radiation into consumable energy necessitate great initial capital costs, which make such investments very expensive for both government and private sector [7]. Once the investment decision is made such systems do not provide reversibility due to the high costs of disassembling, transportation, and reassembling procedures. Thus, from an investor's point of view, due to the irreversibility of the asset and the uncertainties the investment possesses, it is hard to come up with a clear investment decision. Due to the high initial capital costs of this new technology, and uncertain, yet increasing annual costs and high inflation rates, the most common valuation method, Net Present Value (NPV) method falls short because of its deterministic nature and the now-or-never enforcement. Government incentives and subsidies in many countries provide additional financial support to encourage the adoption of solar energy, making it even more accessible to households, businesses, and communities. Still, according to traditional valuation methods, these investments can result in negative cash flows.

The Real Option Valuation (ROV) method is an advanced financial analysis method that evaluates an investment's potential flexibility and strategic value by treating it as an option. The investor might be well equipped with some decision-making options, which might be reevaluated at different points in time when the level of uncertainty is decreased or even eliminated. It is used to analyze and make strategic decisions on investment opportunities with high levels of uncertainty. While this method may not eliminate the entry barrier posed by the high initial costs of solar investments, its ability to probabilistically project uncertainty factors and ensure flexibility through the valuation of different options under uncertainty makes ROV to secure more favorable results. This approach has the potential to make solar power investments economically viable at various points in time. By incorporating the option value, which reflects the value produced with the managerial moves of the decision makers in the procedure, more realistic results may be obtained in favor of solar energy investments.

This study utilizes the ROV method in the Turkish residential solar energy market, assessing investment feasibility and estimating potential government incentive benefits

through a case study. Thus, it demonstrates the applicability of the ROV method in the context of PV rooftop systems, particularly in situations characterized by high economic uncertainty. This method proves to be valuable for small-scale investors, such as homeowners, who are confronted with investment decisions. Furthermore, the research examines the influence of government incentives on feasibility assessments through utilizing a case study. Notably, the study underscores the pivotal role played by government incentives in this scenario. While previous research has explored the contribution of ROV on residential solar panel investments for different countries, this study stands as a pioneering effort, uniquely employing the ROV methodology to analyze the impact of government incentive policies specifically within the domain of residential rooftop PV technology. Addressing a research gap, this study aims to contribute practically to both industry and government, serving as a valuable example for similar conditions in other countries.

The research objectives, methodology and design will be explained in Section 2, particularly elaborating on ROV as an appropriate method to improve decision-making in residential PV investments.

2. Research objectives and methodology

This study aims to propose an effective method to be used during evaluation of residential PV investments, particularly in developing countries where uncertainty is high and government investments are not mature, such as Turkey. Thus, the main objectives of this study are:

- (1) To identify alternative valuation methods to be used for evaluating the attractiveness of PV investments considering uncertainties and alternative managerial actions.
- (2) To apply the relevant method for evaluation of a residential PV investment in Turkey and to demonstrate its possible benefits for evaluation of investments and formulating government incentives.

The research methodology utilized in this study is given in Fig. 1. Literature review was performed on energy investments to realize the importance of the residential PV investments and their valuation methods to identify the possible alternatives. Consecutive interviews with two experts on the field were made to investigate the current status of the residential PV investments in Turkey and their valuation procedures in practice.

“Real Options Valuation with Least-Squares Monte Carlo Simulation (ROV-LSMC) Method” was selected as the appropriate method based on the findings from the literature review and the interviews. The method was applied to a residential PV investment in Turkey with consideration of deferral option.

A literature review was conducted and the international practices on incentives were investigated in pursuit of finding a solution to the entry barrier problem of residential PV investments. Then, additional government incentives, that are in alignment with the literature and the information gathered from the interviews, were investigated within scenario analysis in terms of the benefits that they might create. Research steps are explained in detail in the forthcoming sections.

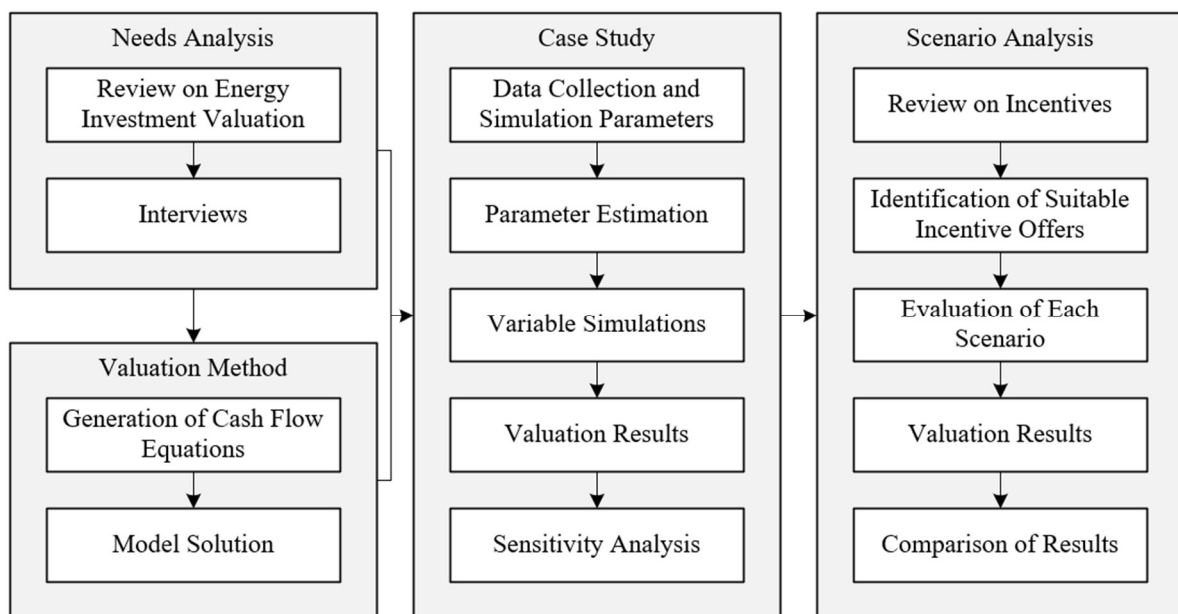


Fig. 1. Research methodology.

3. Needs analysis

This section explores the reasoning that leads the research to the selection of ROV-LSMC method to evaluate residential PV system investment projects and main assumptions made for the case study held.

3.1. Investment valuation approaches

The managerial ability for adapting to changing circumstances and making various strategic decisions over time brought in by real options approach constitutes its major strength against the traditional valuation methods [9]. Even though the traditional methods recognize the uncertainties within the investments, they do not account for how managerial actions can help to mitigate those risks and potentially enhance the value of the project [10]. The discounted cash flow approach assumes a fixed set of outcomes through a more static and single-path decision-making process that yields a now-or-never type of conclusion. In contrast, the real options approach allows for the possibility of multiple pathways and midcourse adjustments based on new information as it becomes available. The real options approach recognizes that there is often a high degree of uncertainty in business situations and flexibility is needed to make the most optimal decisions. Thus, ROV is a more powerful financial analysis tool for investments that include uncertainty. ROV analysis can be made through main three approaches as follows [11]:

(1) Partial differential equation approach is based on generating a numerical conclusion by solving the formed equations for the real options (e.g., Black&Scholes Model) with limitations as not providing the value of the option at all times and lacking transparency [11].

(2) Dynamic programming approach allows for considering a broader range of potential values of the underlying asset over the option's duration and enables decision-making by visualization of the intermediate steps (e.g., Binomial Lattice Model) [11].

(3) Simulation approach tries to evaluate the option price based on a considerable number of simulations from now to the option maturity time (e.g., Monte Carlo Simulation Model) with the drawback of being highly dependent on the input parameters [12].

ROV-LSMC enhances simulation approach with backward dynamic programming approach to evaluate the price of the option. The method is advantageous when compared with the Black&Scholes and Binomial Lattice methods since these methods are not able to consider multiple factors and variables in an investment [13]. Even though the method still possesses the weakness of dependency on the input parameters, the method is said to be powerful since it offers a more robust, intuitive, and easy-to-implement way for solving multiple options with the built-in advantages of simulation approach such as easiness, simplicity, and transparency. In the light of the presented information, ROV-LSMC was selected as the appropriate ROV method for the study.

3.2. Real options valuation of energy investments

The real options approach has been used widely in renewable energy investment valuations and resulted in comprehensive outcomes for such investments [14–17]. The success of the method stems from the similarities between renewable energy investments and options in stock prices. They both have great uncertainty for future prices, the possibility of acquiring better information/estimation about the futures prices as time passes and the chance of postponing (unnecessity of executing the investment or the option in case of American type) exist in both. Thus, the real options approach best fits with reflecting the managerial flexibility for these investments. Table 1 presents the notable studies on ROV oriented solar energy project valuation.

Table 1. ROV studies on solar energy projects.

Study	System	Country	Method	Uncertain Factors
Ashuri et al. (2011) [18]	Rooftop PV systems	USA	Binomial lattice model	Electricity price
Jun et al. (2014) [19]	PV power generation projects	China	Binomial lattice model	Investment cost
Reisi Gahrooei et al. (2016) [16]	Rooftop PV systems	USA	Binomial lattice model	Building demand, electricity price, solar panel cost
Kim et al. (2016) [20]	PV power generation projects	South Korea	Binomial lattice model	Climate
Zhang et al. (2016) [17]	PV power generation projects	China	Least-Squares Monte Carlo	CO2 price, energy cost, investment cost, market price of electricity
Cheng et al. (2017) [21]	PV power generation projects	China	Monte Carlo	PV module costs, electricity prices, support schemes
Kim et al. (2017) [22]	Building integrated PV systems	South Korea	Binomial lattice model	Electricity price
Zhang et al. (2017) [23]	PV power generation projects	China	Least-Squares Monte Carlo	Market price of electricity, CO2 price, investment cost
Li et al. (2018) [24]	PV power generation projects	China	Binomial lattice model	Environmental cost of desertification control, thermal power cost, power generation cost, carbon prices, government subsidy
Moon and Baran (2018) [25]	Rooftop PV systems	USA, Germany, Japan, Korea	Monte Carlo	Investment cost
Penizzotto et al. (2019) [14]	Rooftop PV systems	Argentina	Least-Squares Monte Carlo	Tariffs, investment cost
di Bari (2020) [26]	PV power generation projects	Italy	Binomial lattice model	Tax benefits

Ma et al. (2020) [27]	Residential PV-battery systems	Australia	Least-Squares Monte Carlo	Peak power demand, diesel fuel price, cost of PV-battery technology
Pringles et al. (2020) [15]	PV power generation projects	Argentina	Least-Squares Monte Carlo	Investment cost, net revenue of the investment
An et al. (2021) [28]	PV power generation projects	Korea	Least-Squares Monte Carlo	System marginal price, Renewable energy certificate spot market price, generated solar power
Assereto and Byrne (2021) [29]	PV power generation projects	Ireland	Least-Squares Monte Carlo	Electricity price
Haikal-Leite et al. (2021) [30]	PV power generation projects	Brazil	Monte Carlo	Solar irradiation
Yu et al. (2022) [31]	PV power generation projects	China	Partial differential equation	PV feed-in tariff, PV power generation cost, carbon emission trading mechanism
Biancardi et al. (2023) [32]	PV power generation projects	Italy	Partial differential equation	Investment cost
Yang et al. (2023) [33]	PV power generation projects	China	Least-Squares Monte Carlo	Investment cost, electricity price, carbon price

Table 1 shows that various applications of ROV exist for solar PV investments. There are also various works in Turkey such as Toptaş [34], Kılavuz [12], Kumbaroğlu et al. [35], and Öztürk [36] that used ROV in their studies on various energy investments. Although several studies exist on solar PV investments in Turkey, such as Yalılı [37], Öztürk et al. [38], and Sogukpinar and Bozkurt [39], none of them used ROV to evaluate residential solar PV investments in Turkey. This study handles the commonly used ROV-LSMC method for the Turkish residential solar energy market and tests the feasibility of these investments in this market through the held case study. Benefits of the possible government incentives are estimated by utilizing the method. Thus, the study addresses the identified research gap and aims to provide practical contributions to both the industry and government. It is believed that the study constitutes an example and presents valuable findings for countries with similar conditions.

3.3. Interview findings

Building upon the literature review findings, interviews were conducted with two experts, whose profile is given in Table 2, to finalize the details with the case study and the model. Interviews were held through questions grouped under seven sections given in Table 3. The experts led to valuable findings for the Turkish case.

Table 2. Profile of the experts.

Characteristics	Expert A	Expert B
Profession	Electrical and Electronics Engineer	Civil Engineer
Years of Experience in Solar Energy	10 years	15 years
Current Occupation	Solar Energy Investment Consultancy and Projecting	Solar Energy Investment Consultancy to both Public and Government
Other	Owner of the Company Experienced in implementation of solar PV systems in Turkey Provides consultancy to investments in PV systems at any scale	Participated in establishing the initial regulations for solar energy sector in Turkey

Table 3. Interview questions.

Section	Question
1	What is the current status of residential PV investments in Turkey? What are the reasons of the lack of prevalence of such investments? What are the current incentives in Turkey?
2	How the initial investment cost and future cash flows are generated in the residential PV investments? What are the cost and revenue items? What is the key information for residential PV systems such as their service life, maintenance requirements, etc.? How can the historical market data for these items be obtained?
3	Which currency do you use while calculating the cash flows and preparing feasibility reports?
4	Which methods do you use while making residential PV investment valuations? Have you ever heard of the Real Options Valuation method? What do you think about its applicability to residential PV investments?
5	What is the best scenario that a house owner can achieve to earn the highest profit from the residential PV investment? Is it wise to spend huge amounts of money to construct the biggest possible PV system on the roof?
6	What is the efficiency of solar PV systems?
7	Is it possible to sell solar PV systems once they complete their service life and generate income? Do these systems have a scrap value?

(1) In consideration of the general overview, experts confirmed that residential PV investments have not been very common in Turkey and there exists a great need for such systems as a response to environmental concerns. The reason behind lack of prevalence of these systems were denoted to be lack of knowledge on the systems and their applicability in small scale to rooftops, requirement of high initial investment costs, and difficulty in estimating whether these investments pay off or not. They informed that Turkish government has been offering net metering that allows deductions from payments of the investor and even profits from the excess energy transferred to the grid on a monthly basis.

(2) As the fundamental information for a possible model, experts listed main six initial investment cost items, which are the panel prices, the inverter prices, the hardware and load-carrying system costs, labor costs, and the project design costs. Expert B stated that the PV panels and inverters generate almost 60% of the initial investment cost and they have been the main reasons for the massive investment cost. Experts noted that, for the entire service

life, which is estimated to be 25 years, the system would need a one-time inverter replacement and several small hardware item replacements. Residential roof PV systems do not require additional operation and maintenance costs, and even periodic cleaning due to their small surface areas. However, cleaning the PV panels can lead to a slight increase in the energy production from the panels. Expert A noted that as the size of the system increases, the unit cost decreases. For the revenue generation of the systems, experts informed that it would vary from house to house and from time to time due to dependency on the size of the system, consumption by the house, the amount of generated electricity and the amount of generated electricity transferred to the grid in the relative month. Both experts shared the information regarding several recent investments and their costs for the historical market data. They also suggested several websites for data collection; however, they both confirmed that it could be hard to find accurate historical data for the Turkish market since such investments have been relatively new in Turkey.

(3) For the calculation of cash flows, experts stated that using USD would be much more accurate for the valuation of residential PV investments in Turkey since the prices of panels and inverters have been in USD.

(4) NPV was confirmed as the most common investment valuation method by both of the experts while Expert A informed that he has also used the Rate of Return method. As for the knowledge in the ROV method, Expert A stated that he has never heard of the method but Expert B stated that he has come across that valuation method but has never used it in the valuation of residential PV investments in Turkey. However, after a brief explanation of the method, Expert B stated that using ROV in residential PV investment valuation would be appropriate.

(5) Both experts advised that the installed PV systems should generate electricity that would be equal to the instant demand in order to acquire the maximum possible profit. This is due to the fact that the unit price of selling the electricity is lower than the unit price of buying electricity since there are additional costs included in the electricity bills that are not included when selling. Therefore, Expert B stated that for the case study a hypothetical scenario in which the investor consumes all the electricity generated and does not sell any should be considered to make an accurate estimation of the revenues.

(6) For the efficiency of solar PV systems, Expert A indicated a varying value between 70 to 85%. Since many factors can affect this efficiency, he provided the real electricity production data of a 6.6 kWp residential PV system and suggested use of this data for the case study. Expert B informed that the efficiency of the PV systems is around 80%, which might increase with the technological advancements and production of new technology panels.

(7) Regarding the scrap value, both experts stated that there has not been a certain value since the PV investments in Turkey have not reached to the end of their service lives. However, Expert B stated that the panels, inverters, hardware, and structural elements possess valuable raw materials and they should have a considerable scrap value.

4. Real options valuation method

Fig. 2 summarizes the ROV methodology where the details are presented in the following sections respectively.

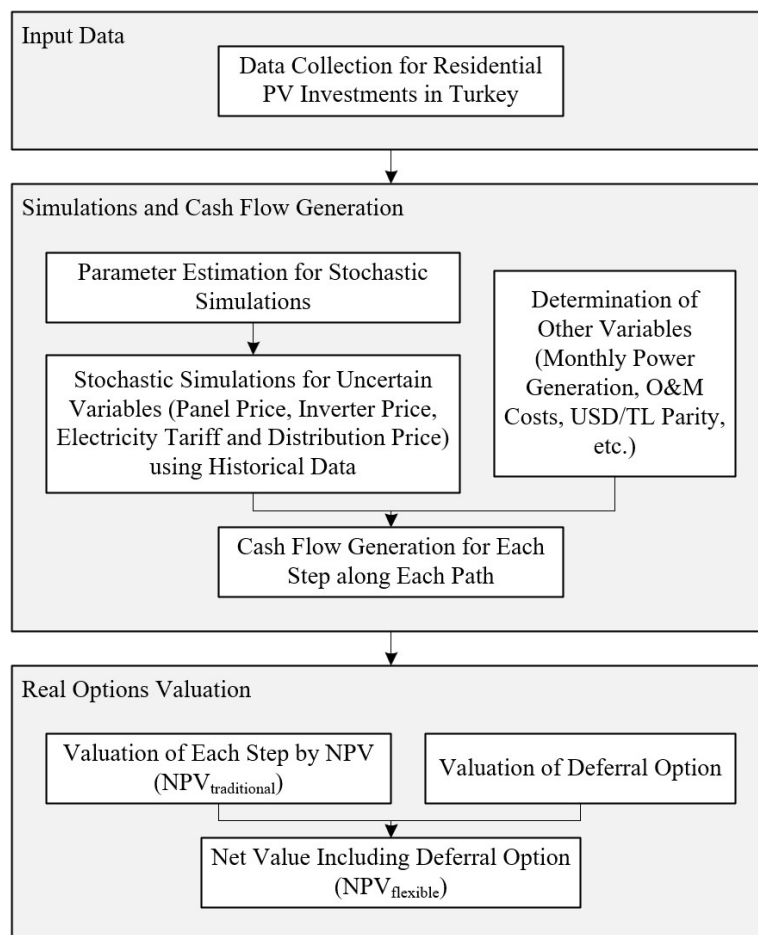


Fig. 2. ROV methodology.

4.1. Investment model

The investment projects shall be first assessed using NPV to calculate the value of the solar power investments using ROV-LSMC. Several assumptions must be made before model

generation to identify the different cost and revenue items that make up the cash flow. Given the power industry's strong dependence on legislation, which is susceptible to significant changes over the years, such assumptions play a crucial role in shaping the model.

4.2. Investment revenues

As identified through interviews to eliminate lower rate of return, the first assumption was made on production capacity for earning the maximum benefit using the PV system. Assumption 1: The instantaneous energy produced at the on-site PV system is always equal to or less than the instantaneous energy demand, and the generated power is 100% consumed, and none is supplied back to the grid.

Secondly, since the generated revenue is actually a reduction in electricity bills, government taxation policy has to be considered. Thus, second assumption was made based on current legislation to neglect the effects of a change in government taxation policy. Assumption 2: Overall tax rate is taken as 18%, which includes several taxes for electricity bills.

Then, by considering the two assumptions, the revenue equation was generated through the formulae explained as follows.

As the initial consideration, the electricity price that a user without a PV system pays at time t :

$$Bill(t) = E_c(t) (P_{trf}(t) + P_{dst}(t)) + tax \quad (1)$$

where $E_c(t)$ is the energy consumed by the house between the instant $t - 1$ and t , $P_{trf}(t)$ is the tariff price of electricity at the instant t , $P_{dst}(t)$ is the cost of electricity transmission from the power plant to the house (i.e., distribution cost) at the instant t . Tax includes all the taxations made by the government in an electricity bill and estimated to be 18%. Thus, the following equation was formed.

$$P_{bill,w.o.PV}(t) = [E_c(t) * (P_{trf}(t) + P_{dst}(t))](1 + tax) \quad (2)$$

The electricity price that a user will pay with a PV system at time t :

$$Bill(t) = (E_c(t) - E_g(t))(P_{trf}(t) + P_{dst}(t)) + tax \quad (3)$$

where $E_g(t)$ is the energy generated by the house and $E_c(t)$ is the energy consumed by the house between the instant $t - 1$ and t . Considering the assumptions made, the total

demand for the house from the grid will decrease with respect to the generated power by the panels, and the user will be charged accordingly.

$$P_{bill,w.PV}(t) = \left[(E_c(t) - E_g(t)) (P_{trf}(t) + P_{dst}(t)) \right] (1 + tax) \quad (4)$$

Then, the final case was subtracted from the initial case in order to calculate the benefit that the solar investment generates.

Revenue (Initial Case – Final Case):

$$R(t) = P_{bill,w.o.PV}(t) - P_{bill,w.PV}(t) \quad (5)$$

$$R(t) = \left[E_g(t) (P_{trf}(t) + P_{dst}(t)) \right] (1 + tax) \quad (6)$$

The corresponding values of the variables in the equation for each month have to be determined to calculate the revenue for each month during the investment period. There exist many different approaches for modeling the electricity generated each month. The main determinants in the power generation of solar PV systems are solar radiation, temperature, clearness index of the area where the system is located [39]. Also, the efficiency of the PV panels and inverters is important while modeling the monthly power generation of the installed system. For this model, the monthly power generation rates of a real-life PV System located in Kocaeli, Turkey, were used to overcome the uncertainties generated by the variables [40].

The future electricity tariff and distribution price is another major uncertainty that has to be modeled in order to be used in the revenue equation. Even though the prices are shown as separate items in an electricity bill, their pricing is heavily dependent on each other. Thus, they were assumed as a single price by their summation in this study. The most commonly used approach to model the stochastic behavior of the tariff and distribution price is the Geometric Brownian Motion (GBM) [14,16,17,26,35,41,42]. Hence, the value of the electricity tariff and distribution price at any time t is given by:

$$P_{et\&d}(t) = P_{et\&d}(t_0) \exp[(\alpha - \sigma^2/2)t + \sigma W_t] \quad (7)$$

where $P_{et\&d}(t_0)$ is the initial electricity tariff and distribution price at t_0 , α is the growth rate of electricity tariff and distribution price, σ is the volatility or the standard deviation of growth rate, W_t is a Wiener process with a mean equal to zero and variance equal to one, $W_t = \sqrt{t} \cdot \varepsilon$ where $\varepsilon \sim N(0,1)$.

Since the electricity tariff and distribution prices in Turkey are in Turkish Liras, and the proposed method aims to make the valuation in USD, the USD/TL parity among the investment's service life has to be determined. A deterministic approximation was decided to be suitable for identification of the future values.

4.3. Investment costs

The cost items for a residential PV system investment are the system hardware costs, direct labor costs, indirect labor costs, permit-inspection-interconnection costs, overhead costs, and sales and marketing costs as stated in US Solar Photovoltaic BESS System Cost Benchmark Report for Q1 2021 [43]. The system hardware costs consist of panel costs, inverter costs, structural balance of system costs, and electrical balance of system costs. Since the panel and inverter costs of PV systems are equal to 60% of the whole investment cost [17] and the drivers of uncertainty regarding the future investment cost of PV systems, they should be investigated in depth. The direct labor costs include electrical, mechanical, and general construction labor costs. The labor costs also vary over time, but they were modeled as a deterministic process for this study due to their low impact on the total investment cost. The indirect labor costs are engineering design and construction permit administration. For residential PV system investments, there is no requirement for either an engineering or a construction permit. Thus, these items were not taken into consideration. The permit-inspection-interconnection costs are self-explanatory and similar to previous cost items; these costs were not taken into consideration since residential PV investments in Turkey do not have such cost items. Finally, overhead and sales, and marketing costs were not taken into consideration in accordance with Assumption 1 since there will not be a case of selling the excess electricity generated by the PV system.

The panel prices and inverter prices have been decreasing heavily as technology advances [44]. A stochastic model has to be used following the real options approach in order to model this uncertainty for the future costs of both these items. However, both for PV panel costs and inverter costs the decreasing trend is not uniform and faces price jumps over time which causes instant price escalations or declinations. Thus, this has to be considered in the model as well. For stock prices that face price jumps similar to the PV panel prices, the GBM falls short. Thus, Merton Jump Diffusion Model (MJDM) model was applied and brought comprehensive results [45]. Finally, the GBM with the Poisson events model was used for the description of the impact of radical technology innovation on the market [46]. R&D processes and product innovations are the main causes of the decrease in the costs of PV panels and inverters. Thus, PV panel and inverter costs can be treated as a part of radical technology innovation. Finally, the uncertainty in PV panel prices and inverter costs were modeled using the MJDM, which was shown to be suitable in the work by Penizzotto et al. [14]. Thus, the value of PV panel cost and inverter cost at any time t are given by the following same formulae:

$$C_{pan}(t) = C_{pan}(t_0) \exp[(\alpha - \sigma^2/2 - \lambda k)t + \sigma W_t + \sum_{i=1}^{N_t}(V_i)] \quad (8)$$

$$C_{inv}(t) = C_{inv}(t_0) \exp[(\alpha - \sigma^2/2 - \lambda k)t + \sigma W_t + \sum_{i=1}^{N_t}(V_i)] \quad (9)$$

where $C_{pan}(t_0)$ is the initial cost of panel at t_0 , $C_{inv}(t_0)$ is the initial cost of inverter at t_0 , α is the growth rate of panel/inverter cost and σ is the volatility or the standard deviation of growth rate without considering the Poisson jumps. λ is the mean number of arrivals per unit time, k is equal to $E[V_i - 1]$ where $(V_i - 1)$ is the random variable percentage change in panel cost if the Poisson event occurs. The value of k is calculated by $e^{\mu_j + 1/2\delta^2}$ where μ_j is the expected value of the jump size and δ is the standard deviation of the jump size. W_t is a Wiener process with a mean equal to zero and variance equal to one, $W_t = \sqrt{t} \cdot \varepsilon$ where $\varepsilon \sim N(0,1)$. $\sum_{i=1}^{N_t}(V_i)$ is the compound Poisson process which is equal to zero when there is no Poisson event between t_0 and t , V_i resemble the jumps, which are independent of the Wiener process. Overall, the α, σ and W_t variables resemble the continuous GBM and $\lambda, k, \sum_{i=1}^{N_t}(V_i)$ variables resemble the Poisson events that cause the jumps of panel costs. The parameters for both the PV panel cost and inverter cost were estimated in accordance with the offered method in Özdemir [47].

The other initial investment costs, such as labor costs, other hardware costs, and load-carrying structure costs, were modeled using a deterministic process where collected previous data was investigated and a reduction rate was estimated. The equation for other costs was formed as,

$$C_{oth}(t) = C_{oth}(t_0)(1 - O_{r.rate})^t \quad (10)$$

where $C_{oth}(t_0)$ is the initial value of other costs at time t_0 , and $O_{r.rate}$ is the reduction rate. Then, the overall initial investment cost of the residential PV system including a taxation of 18% can be calculated by,

$$I(t) = [C_{pan}(t) + C_{inv}(t) + C_{oth}(t)](1 + tax) \quad (11)$$

4.4. Operation, maintenance, and disposal costs

As identified through interviews, residential PV systems were assumed as they do not require a periodic maintenance service unless there is a malfunction. Additionally, rainfalls were assumed to be sufficient for the removal of dust and dirt over the PV panels due to their small surface areas, which eliminated the yearly routine cleaning requirement. Thus, the yearly operation and maintenance cost includes the replacements costs (for inverter, module and component parts), system inspection and monitoring costs, and insurance [43].

Since the total cost of operation and maintenance is relatively low compared with the PV panel and inverter prices, similar to other initial investment costs, they were modeled using a deterministic process through an estimated reduction rate using the historical data. The equation for yearly operation and maintenance cost was defined as,

$$C_{O\&M}(t) = C_{O\&M}(t_0)(1 - O_{O\&M,r.rate})^t \quad (12)$$

where $C_{O\&M}(t_0)$ is the initial value of operation and maintenance cost at time t_0 , and $O_{O\&M,r.rate}$ is the reduction rate.

For the disposal costs, since the PV system investments are relatively new, with an expected service life of around 20 to 25 years, there has not been any information about the disposal cost of PV systems in Turkey [38]. Therefore, the residential PV system was assumed to have no disposal costs or salvage values.

4.5. Model solution

The traditional NPV (NPV_{trd}) of the residential PV investment at any time t using the revenues and costs identified was calculated by,

$$NPV_{trd} = -I(t) + \sum_{t=1}^{T_{life}} [R(t) - C_{O\&M}(t)] / (1 + i)^t \quad (13)$$

where $I(t)$ is the initial investment cost at time t , $R(t) - C_{O\&M}(t)$ is the revenue function for any time step between the initial investment time and the service life of the investment, T_{life} , i is the discount rate determined by the opportunity cost of capital.

The values of $I(t)$ and $R(t)$, which are the initial investment cost and revenue function, respectively, were estimated using the values of PV panel and inverter costs and electricity tariff and distribution costs. Since the future prices of these items contain too much uncertainty, they can only be modeled using a stochastic model in order to determine the prices during the investment period. The offered methods simulate a number of various paths using the input parameters and the resulting NPV_{trd} values was calculated for each path. Then the expected value of the NPV_{trd} values generated from different paths, $E(NPV_{trd})$, was calculated as the average of these NPV_{trd} values. Also, the standard deviation of these NPV_{trd} values was calculated as, $\sigma(NPV_{trd})$.

Even though this approach handles the uncertainty of items used in the NPV_{trd} calculations, managerial flexibility, such as the deferral of the investment time, is not considered. In the deferral option, the investor has the flexibility of postponing the decision to invest in the project in pursuit of getting a clear view of the investment and its cost items. This

deferral option is very similar to the American call option, in which the option holder has the privilege to acquire the stock at the cost of the strike price at any time during the maturity time, the time frame for which the option is valid. For a residential PV investment, the value of the option to defer the investment at any time t and a specific path w can be calculated using the same formula for calculating the value of an American call option as follows [14],

$$F(t, w) = \max_{\tau \in Y(t, T)} \{ \mathbb{E}_Q [e^{-r(\tau-t)} \Pi(\tau, X_T)] \} \quad (14)$$

where $Y(t, T)$ is the set of optimal times to exercise the option during the defined time frame $[t, T]$. T is the option maturity, the latest time the option is valid. $\mathbb{E}_Q[\cdot]$ is the risk-neutral expected value operator which is subject to the information set available in time t and the revenue function, $\Pi(\tau, X_T)$, for the option at time instant τ .

The formula can be solved using the Least-Squares Monte Carlo (LSMC), which offers an approximate path for the solution of the stopping problem generated by the American call option. In the LSMC evaluation, w paths have to be generated, which will be used for the simulation of the stochastic dynamics of the state variables X_T that affect the value of the option. Then, the evaluation begins at the option maturity date and continues recursively until t_0 working towards the generation of an exercise rule that maximizes the option value at each time step t along the generated w paths. At time T , the option maturity, the value of exercising the option at T , is compared with the value of the underlying asset. If the value of the cash flows generated by making the investment is greater than the value of the final deferral decision for investment which results to zero, then the option is exercised. Thus, the optimal option value at option maturity was calculated as follows,

$$F(T, w) = \max[-I(T, w) + PV(T, w); 0] \quad (15)$$

where $F(T, w)$ is the value of the call option at time T along path w and $PV(T, w)$ is,

$$PV(T, w) = \sum_{t=T}^{T+T_{life}} [R(t, w) - C_{O\&M}(t, w)] / (1 + i)^t \quad (16)$$

For any time, t_i previous to the option maturity date, the optimal strategy to execute results from comparing the value of cash flows generated when the investment is exercised at t_i versus the expected value of the cash flows that might happen by continuing, i.e., keeping the option alive. If the value of immediate exercise is greater than the value of expected cash flows that might arise when continuing, the investment is exercised.

$$F(t_i, w) = \max[-I(t_i, w) + PV(t_i, w); \emptyset(t_i, w)] \quad (17)$$

The value of continuation, $\emptyset(t_i, w)$, has to be determined to find the value of option. In the theory of arbitrage free valuation, the value of continuing is determined by expectation of

the cash flows generated by the option $F(t_{i+1}, w)$ discounted with respect to a risk-free measure Q , where r being the risk-free discount rate.

$$\emptyset(t_i, w) = (1 + r)^{-(t_{i+1}-t_i)} \cdot \mathbb{E}_Q[F(t_{i+1}, w)] \quad (18)$$

Since the goal of LSMC is the maximization of the option value, this can only be achieved once the decision to exercise the investment is made when the immediate value of exercise is greater than the value of continuation. Thus, the whole offered model leans on the correct estimation of the continuation value. At each time instant t , the LSMC utilizes the least squares regression technique to approximate the conditional expectation function, $\emptyset(t_i, w)$. The conditional expectation functions at each time instant t are represented as a linear combination of a countable set of orthonormal basis functions $\{LM\}$. The most common function used are Laguerre, Hermite, Legendre, Chebyshev, Gegenbauer and Jacobi polynomials [13].

$$\emptyset(t_i, w) = \sum_{m=1}^{\infty} \varphi_m(t) \cdot L_m(t, X_m) \quad (19)$$

For the estimation of the values of φ_m , the least square regression of $\emptyset_M(t_i, w)$ with M elements of the selected base function is used with $M < \infty$ [48].

$$\{\hat{\varphi}(t_i)\}_{m=1}^M = \arg \min_{\{\varphi\}_{m=1}^M} \left\| \sum_{m=1}^M \varphi_m(t) \cdot L_m(t, X) - \sum_{m=1}^M (1 + r)^{-(t_{i+1}-t_i)} \cdot F(t_{i+1}, \cdot) \right\| \quad (20)$$

where $\|\cdot\|$ is the norm of Hilbert vector space from which the estimated value of the continuation function results,

$$\hat{\varphi}_M(t_i, w) = \sum_{m=1}^M \hat{\varphi}_m(t_i) \cdot L_m(t_i, X_m) \quad (21)$$

While determining the estimated value of continuation function, only the cases that are in the money such as the cases that the value of the underlying asset is greater than the strike price is considered. This is because the decision of exercising the investment or the option is available at such conditions. For the case of the value of the underlying asset is lower than the strike price, there is no point for investor to make a decision since there is not a case that will generate profit. By eliminating the cases that are out of money, the number of base functions required to obtain a good estimation of continuation function is reduced and the approximation is restricted to a much relevant region [15].

When the estimation of the continuation function is done, the decision of exercising the investment or the option can be made easily. If the condition of the immediate exercise value of investment is greater than the value of expected cash flows that might arise when continuing is satisfied, the investment is exercised.

$$[-I(t_i, w) + PV(t_i, w)] > \widehat{\varphi}_M(t_i, w) \quad (22)$$

Once the decision for time instant t_i is made, one can move on with the instant t_{i-1} since the choices are made for t_i and cash flows are generated for that time instant at all paths. This backward recursive process is done until t_0 . By completing this process, the optimal investment timing for each path generated is determined. Finally, the estimated deferral option for the investment is calculated by discounting the option values obtained in each path to t_0 using the risk-free rate, r , and taking their average.

$$F(0) = 1/W \sum_{w=1}^W (1+r)^{-\tau(w)} F(\tau, w) \quad (23)$$

The option value obtained for each path is added to NPV_{trd} value of them and then the $NPV_{flexible}$ (NPV_{flex}) values of each path that includes the value of the deferral option can be estimated.

$$NPV_{flex}(w) = NPV_{trd}(w) + F(t_0, w) \quad (24)$$

When the average value for the paths generated are taken, the expected values for the investment can be estimated.

$$E(NPV_{flex}) = E(NPV_{trd}) + F(0) \quad (25)$$

The standard deviations of the variables were computed as well to verify the model. Also, a sensitivity analysis was made for verification of the offered model.

5. Case study: The residential PV investment in Turkey

The planned residential PV investment is to be constructed in Gebze, Kocaeli, Turkey on the roof of a 3-story building which accommodates three families of four. The investment is composed of 22 panels which covers a 48 m² area on the roof of the building with the estimated capacity of 6.6 kWp. The annual energy production from the system is expected to be 7600 kWh. According to Chamber of Electrical Engineers in Turkey, the monthly consumption of a family of four is 230kWh. Thus, the offered system aims to compensate the 91% of the total power consumed at the 3-story building with an expected 85% efficiency. The service life of the PV investment is considered to be 25 years with a deferral option of investment for 7 years.

5.1. Case background

Being located at the Marmara region of Turkey, Kocaeli is one of the cities with the lowest solar energy potential. Accordingly, conducting the analysis in this city enables a cautious approach and ensures that the results of the study are conservative, which further leads robustness of the model across various levels of solar potential. Moreover, availability of real-world data specific to this city provides mitigation of additional uncertainties due to unreliability of data.

For the simulations and valuations, a self-written program in Python 3 language was used. At first part of the code, the simulations for the variables were completed. The parameters estimated from the historical data was used as input and the simulation results were obtained. In the valuation process, the equations generated served as inputs, and the NPV values for exercising the investment at each time step along the generated paths were calculated. Subsequently, these values were utilized as inputs for the LSMC algorithm to derive the final results.

Instead of relying on estimated values for the monthly energy production of the proposed PV system, actual energy production values from a real case study situated very close to the selected investment area were utilized. This approach was adopted to minimize additional uncertainty within the model. In a similar setup, a total annual energy production of 7578 kWh was achieved. Fig. 3 presents the used monthly histogram of energy production of the real case study.

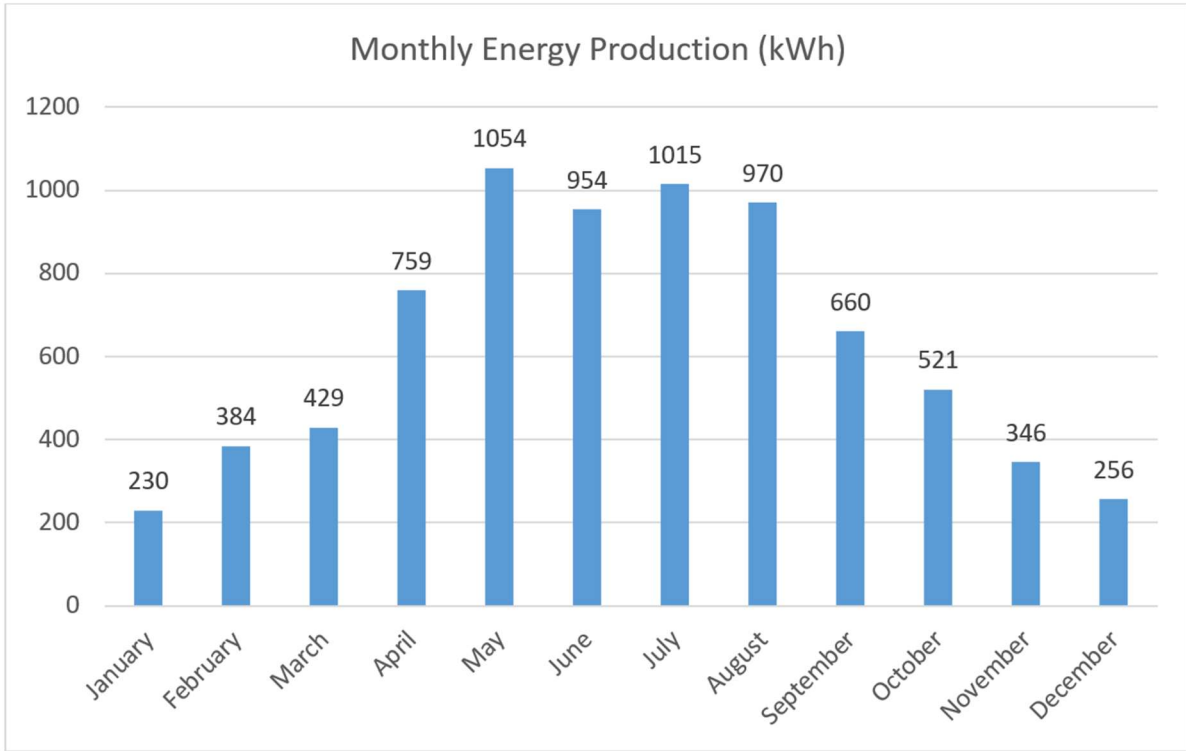


Fig. 3. Monthly energy production of a near PV system investment with a similar setup.

5.2. Data collection and simulation parameters

For the case study, the variables in Equations (1) to (12) such as; $E_g(t)$, $P_{et\&d}(t)$, $C_{pan}(t)$, $C_{inv}(t)$, $C_{oth}(t)$, $C_{O\&M}(t)$, w , i , T_{life} , T , r , $O_{r.rate}$, and $O_{O\&M,r.rate}$ have to be determined. The t values at the variables represents the instants of investment decision points. For this study, it was assumed that at the first day of every month the decision to exercise the investment would be made. Thus, the time step was monthly and the values of time-dependent variables were estimated for each month through stochastic simulations.

In the generation of stochastic simulations such as GBM, it is imperative to estimate the parameters along with determining the number of simulation paths. In Fig. 4, the option value versus the number of simulations is provided, which depicts that using 10,000 is adequate for this study.

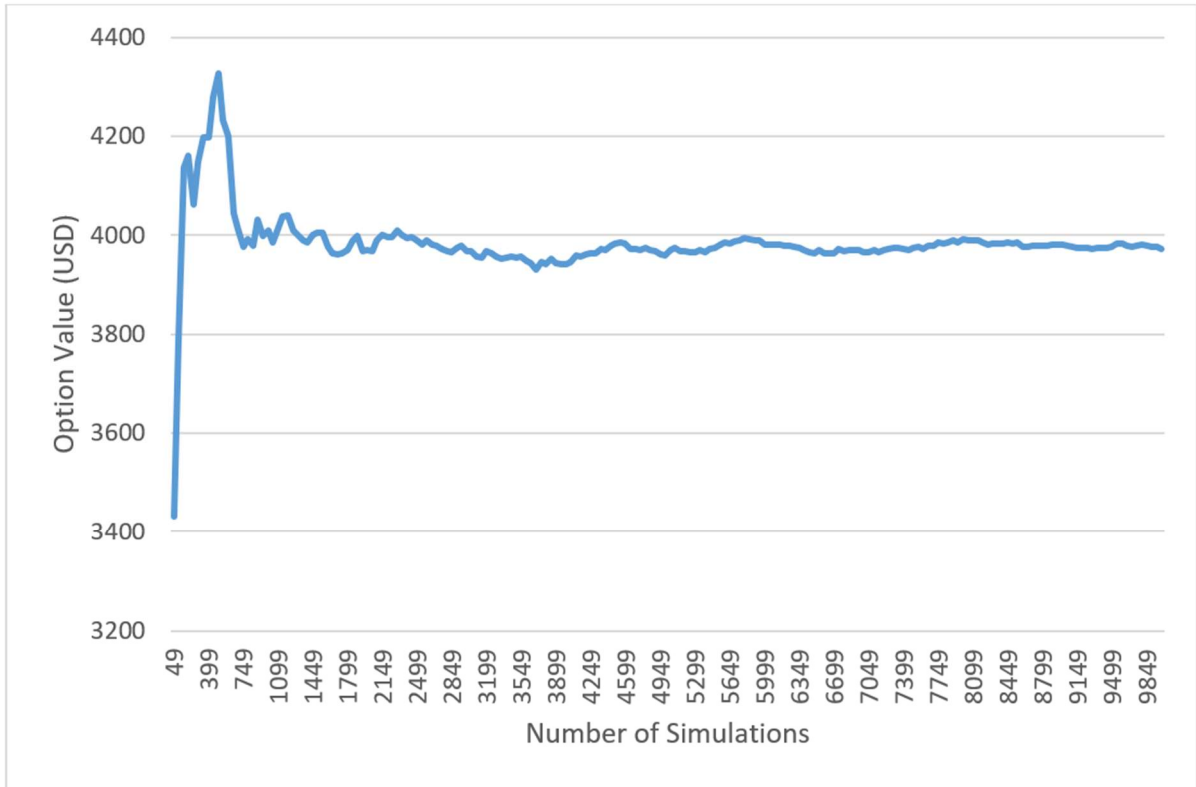


Fig. 4. Graph of number of simulations versus option value.

Table 4 presents the historical data found for the determination of simulation parameters. Even though the monthly generated energy from the offered residential PV system is a time dependent variable, use of the production values from the nearby real PV system with identical attributes eliminated the need of historical data.

Table 4. Historical data for costs.

Cost	Data Source	Range	Remark
Electricity tariff and distribution costs	Monthly real-time electricity bills of a house in Turkey	2011 - 2021	Bill is divided to 1.18 to remove the presumed value-added tax from the total price
PV panel costs	Reports released by the U.S Energy Information Administration [49,50]	1988 - 2021	No historical data source for Turkey
Inverter costs	A study about the “Current and Future Costs of Photovoltaics” and the report released by U.S Energy Information Administration [51,52].	1990 - 2021	No historical data source for Turkey
Total of other PV investment costs and operation and maintenance costs	The U.S Energy Information Administration report [52]	2020-2021	No historical data source for Turkey, a deterministic process was used where the reduction rate was determined to be 1% each year

The rest of the variables that are not dependent on t were determined based on previous studies. The value of i , the discount rate determined by the opportunity cost of capital, was taken between 10-13% in previous studies on renewable energy investments in Turkey [12,34,38] where in the study on the valuation of commercial PV investments in Turkey [37] it was taken as 10%. Due to the course of time, discount rate was identified as 11% for this study. The service life of the PV investment was taken as 25 years as it was identified in the previous studies [14,15]. The option maturity was identified as 7 years and the risk-free discount rate was taken as 8%.

An estimation for the future prices of USD/TL was made by the obtained parity values between 2012-2021 from the Central Bank of the Republic of Turkey [53] database to convert the electricity tariff and distribution price to USD/kWh unit. A linear fit was made (by removing the data to eliminate the jump between September 2021 and December 2021 which disrupted the general linear increase trend in the USD/TL parity) and the slope was determined as 0.06. Then the new trend line was put on top of the value of December, 2021 and the future values of the USD/TL were obtained as shown in Fig. 5. Table 5 summarizes the final simulation parameters used in the case study.

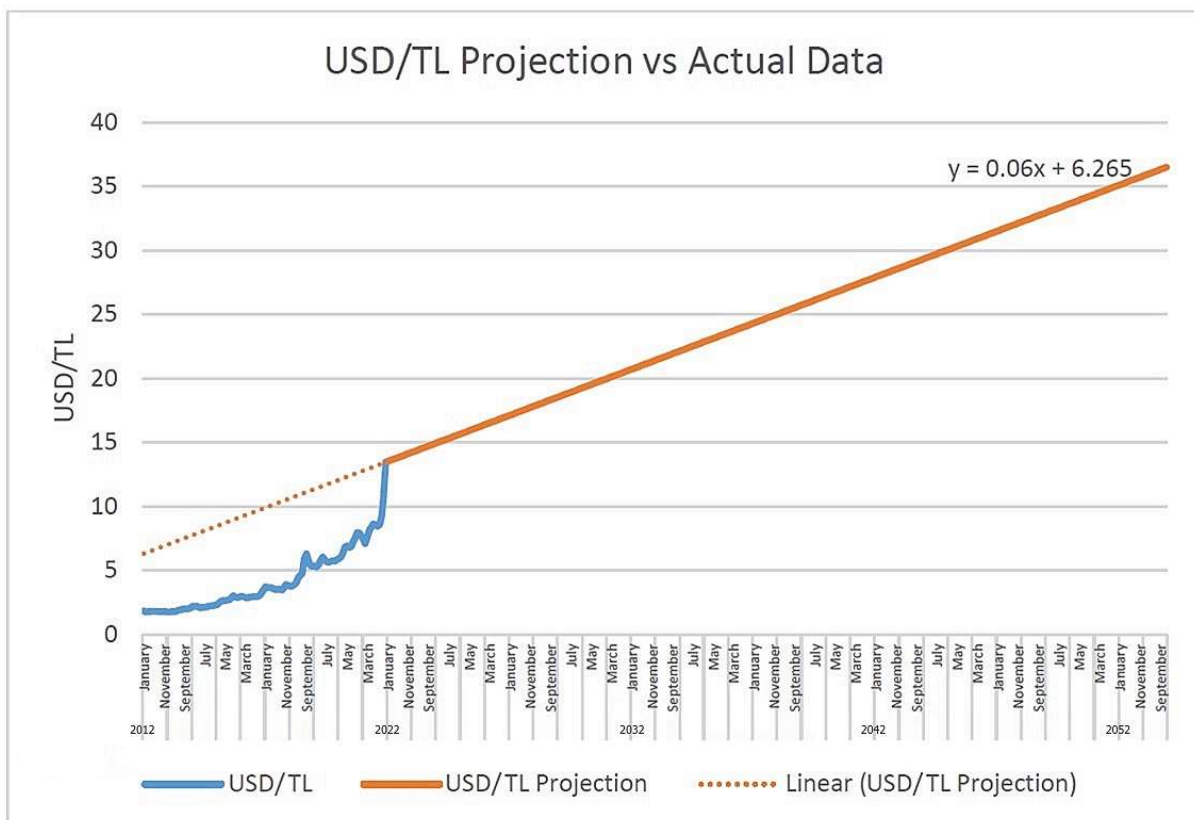


Fig. 5. USD/TL projection vs actual data between 2012-2053.

Table 5. Simulation parameters of the case study.

Parameters	Value	Unit
Capacity of the PV system	6.6	kWp
Annual energy generation of the PV system	7578	kWh
Service life of the PV system	25	years
Deferral option length	7	years
Number of simulation paths	10,000	
Discount rate determined by opportunity cost of capital	11	%
Risk-free discount rate	8	%
Electricity tariff and distribution price at t_0	0.7759	TL/kWh
PV panel cost at t_0	0.3476	USD/Wp
Inverter cost at t_0	0.27	USD/Wp
Total of other initial investment costs at t_0	0.5	USD/Wp
Yearly reduction rate of other initial investment costs	1	%
Operation and maintenance cost at t_0	28.97	USD/Wp
Yearly reduction rate of operation and maintenance cost	1	%
Investment cost at t_0	8703.9	USD

5.3. Parameter estimation

The values of growth rate and volatility were calculated by Microsoft Excel using the historical data (Table 4) to use GBM for electricity tariff and distribution price simulation according to Equation 7. The algorithm proposed by Özdemir [47] was used for the required parameters of the MJDM in simulation of future values of panel and inverter costs. Initially, arbitrary variables were given to the algorithm and the resulting parameters were obtained as provided in Table 6.

Table 6. Parameter estimation results.

Parameters	Growth Rate (α)	Volatility (σ)	Expected Jump Size (μ_j)	Standard Deviation of Jump Size (δ)	Jump Intensity (λ)
For Electricity Tariff and Distribution Price	0.1132	0.1024	-	-	-
For PV Panel Cost	-0.0743	0.1243	-0.0029	0.1243	0.2000
For Inverter Cost	-0.0563	0.0548	-0.0765	0.1289	0.1330

5.4. Variable simulations

By using the obtained parameters, the future monthly prices of electricity tariff and distribution price, PV panel cost and inverter cost were estimated using 10,000 paths. Simulations with 10 paths were done within the timeframe of historical data in order to verify

the parameters and the resulting graphs and expected values at the end of simulations were compared with the actual data.

Fig. 6 exemplifies the output of typical procedure handled for variable simulations. The electricity tariff and distribution price in December, 2011 was 0.2525 TL/kWh. The simulated paths and the real path were obtained very similar to each other as seen from Fig. 6(a). The expected value of the simulation in December 2021 is equal to 0.7397 TL/kWh while the corresponding value in the historical data is 0.7759 TL/kWh. There is only a 5% difference between the simulation results and the historical data. As the number of simulation paths increase, this difference will become much less. Thus, it can be said that the estimated parameters are fitting. A sample from the simulated monthly electricity tariff and distribution prices with 20 synthetic paths is available in Fig. 6(b).

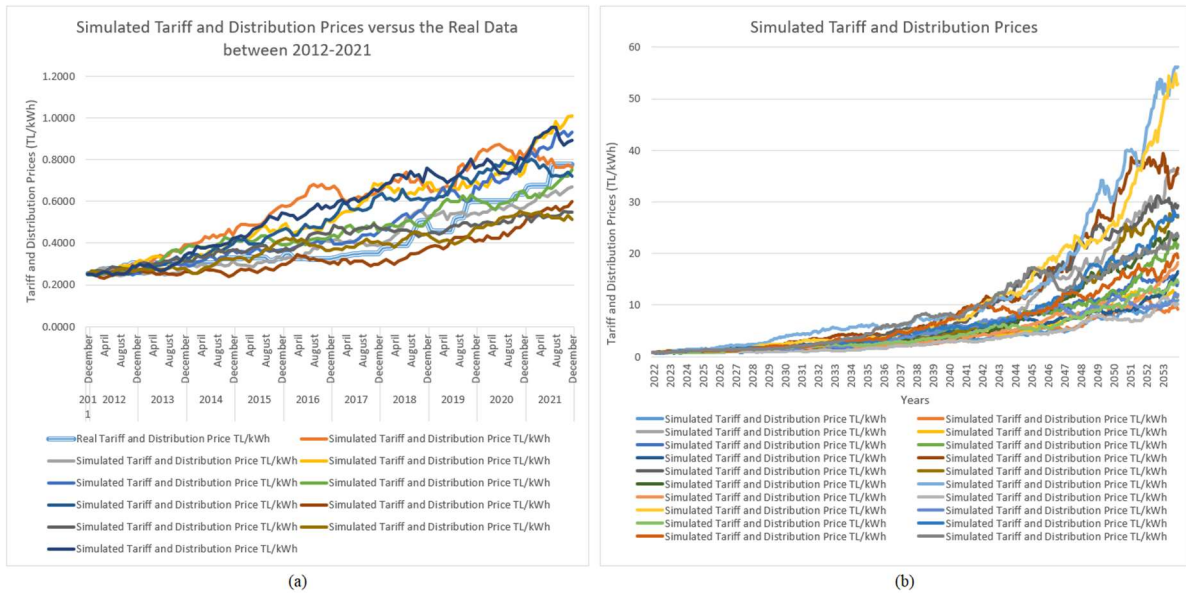


Fig. 6. (a) Simulated tariff and distribution prices versus the real data between 2012-2021 (b) Simulated future tariff and distribution prices.

5.5. Valuation results

The ROV was conducted and the expected values with each of the valuation method were obtained as follows together with the histogram showing the probability density functions given in Fig. 7.

$$E(NPV_{trd}) = -1730.04 \text{ USD}$$

$$E(NPV_{flx}) = 2242.87 \text{ USD}$$

$$E(F(0)) = 3972.92 \text{ USD}$$

As seen from the expected values, the NPV_{trd} results in a negative value, meaning that the investment is currently not desirable. However, when the investor owns a deferral option for the time of investment, an additional value is generated which is denoted as $F(0)$, the option value. By using the proposed method which accounts for this option value at the NPV calculations, the resulting NPV_{flx} results in favor of the investment which means that instead of rejecting the investment today, if the investor postpones the decision making to a future point in time, there exists a possible favorable outcome for the investor.

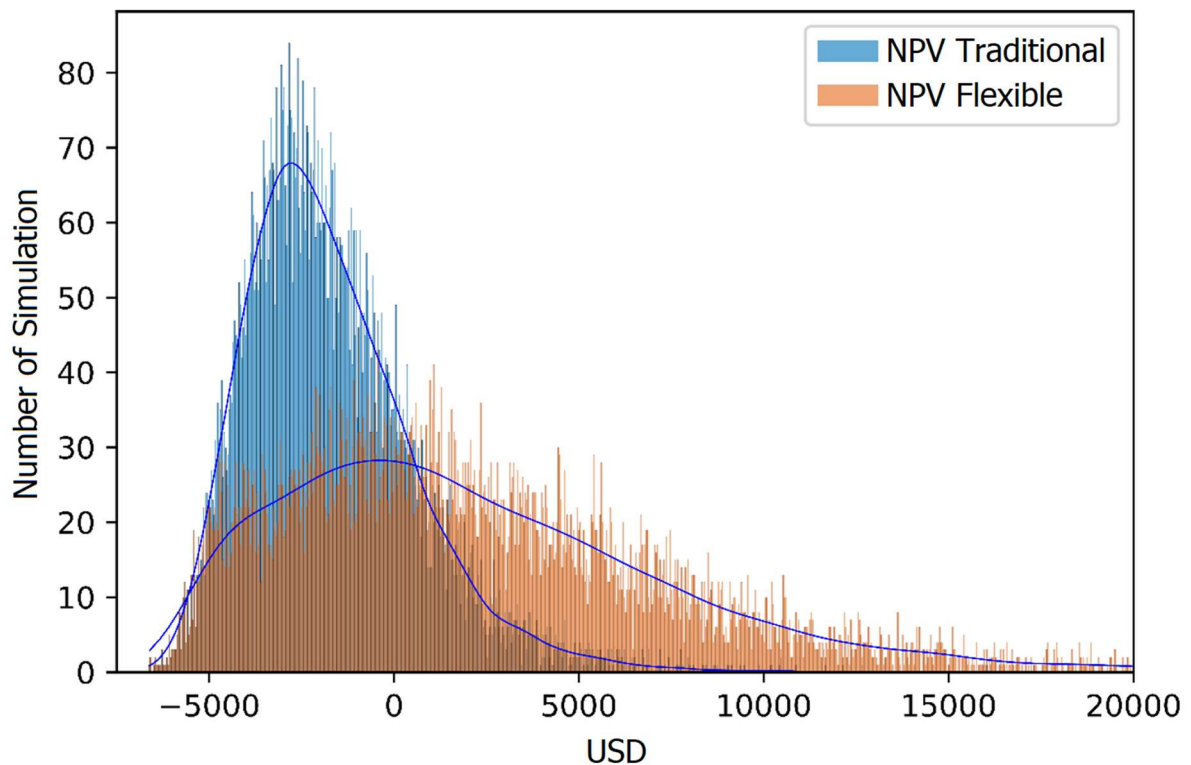


Fig. 7. NPV_{trd} and NPV_{flx} histogram bars.

According to Fig. 7, since most of the blue bars, namely paths of NPV_{trd} , are located at the negative region, it can be said that most of the paths of NPV_{trd} calculation results in negative cash flows. Also, they are almost stacked up between -5000 USD and 2500 USD values, forming a bell-shaped histogram, meaning the probable outcomes from NPV_{trd} calculations are limited in between those values. However, the histogram of NPV_{flx} is right-skewed, namely the NPV_{flx} paths are dispersed towards the right of the x-axis, meaning there are more possible scenarios for the outcomes of NPV_{flx} values. Even though there are negative NPV_{flx} values, the expected value becomes positive due to the skew.

5.6. Sensitivity analysis

Sensitivity analysis was made to evaluate the integrity of the model generated and to identify the impacts of the variables in the generated cash flow equations to the output of the ROV. The time-dependent variables were excluded from the sensitivity analysis since they were simulated using either stochastic or deterministic methods. Also, from the literature review and interviews, it is known that other costs (i.e., labor and operation and maintenance costs) have minimal effect on the outcome. Considering the impact of the number of paths generated in the stochastic simulations it was proven that increasing the value more than 10,000 would not have any impact. Similarly, the service life of the investment, T_{life} , was chosen to be constant with respect to the interview findings. Thus, the variables i , the discount rate determined by the opportunity cost of capital, r , the risk-free discount rate used in LSMC, and T , the value of option maturity were evaluated under 4 different cases. The analysis resulted with the following findings:

- The discount rate, i , and the NPV_{flx} value are inversely proportional,
- The risk-free discount rate, r , and the NPV_{flx} value are inversely proportional,
- The option maturity, T , and the NPV_{flx} value are directly proportional, and
- The impact of the change in the values of i , the discount rate determined by the opportunity cost of capital on the NPV_{flx} is higher than the impact of the r , the risk-free discount rate.

5.7. Scenario analysis

Based on literature review on government incentives for solar PV investments two new government incentives suitable for residential PV investments in Turkey were generated [54]. The two incentives were evaluated in separate scenarios applied to the case study.

Scenario 1: Long Term Low Interest Loan Specialized for Residential PV Investments

For the offered loan, the interest rate was chosen to be 1.12%. It was assumed that the given loan has similarities to the housing loan due to considerably long credit period. The credit period of the loan was assumed to be equal to the service life the investment (i.e., 25 years). The initial investment cost was avoided since the investor would pay this cost using the loan and instead of the requirement of having that whole amount of money at t_0 , the loan repayments were distributed monthly during the service life of the investment.

By using the loan, the equation of the traditional NPV of the residential PV investment at any time t using the identified revenues and costs becomes,

$$NPV_{trd} = \sum_{t=1}^{T_{life}} [R(t) - C_{O\&M}(t) - C_{loan}]/(1+i)^t \quad (26)$$

where C_{loan} is the loan repayment at time t , $R(t) - C_{O\&M}(t) - C_{loan}$ is the revenue function for any time step between the initial investment time and the service life of the investment, T_{life} , i is the discount rate determined by the opportunity cost of capital.

Scenario 2: Cash Rebate Equal to the Tax of the Initial Investment Cost

The second offer aims to decrease the initial investment cost. While making a solar PV investment in Turkey, the investor will eventually pay the designated amount of tax for the initial investment cost. By offering a cash rebate which is equal to the tax of initial investment cost, some part of this high initial investment cost can be diminished. Based on the information gathered from the interviews, solar PV investors are subject to 18% VAT and through the offered cash rebate the investors will retrieve equal amount of money once they make the investment. It is assumed that this retrieval of tax amount is done within the same month of investment. Thus, within the generated cash flows, the initial investment cost and this retrieval will be on the same month.

The following new initial investment cost equation was formed with this offered incentive, where the 1.18 multiplier was removed from the equation. The new $I(t)$ term was used in the generated NPV equations.

$$I(t) = [C_{pan}(t) + C_{inv}(t) + C_{oth}(t)] \quad (27)$$

Analysis Results: According to the adjusted formulae for both scenarios, the offered ROV method was applied. The expected results were obtained as in Table 7 in comparison with the base case, together with the parallel changes obtained in histograms.

Table 7. Scenario analysis results.

Expected Results (USD)	Base Case	Scenario 1	Scenario 2
E(NPV _{trd})	-1730.04	-3337.22	-402.33
E(NPV _{fix})	2242.87	14.17	4116.27
E(F(0))	3972.92	3351.39	4518.60

6. Discussion

The valuation of investment project in the case study with the traditional NPV method yielded to rejection of the project with a value of –1730.04 USD while with the ROV method it turned out to be 2242.87 USD. Thus, the ROV method generated an option value of 3972.92 USD for the investment. The ROV method brings much more comprehensive results since the uncertain variables are handled with stochastic simulations and investors' opportunity to decide later on has been preserved.

The investor's allowed deferral time to make the investment is the option maturity and it is one of the key values in the generated ROV model. The sensitivity analysis resulted that "option maturity" is directly proportional with the results of the ROV method. This means that as allowed deferral time of the investment increases, the results generated from the ROV method will increase. In parallel with this finding, when the current trend of the residential PV investments in Turkey is investigated, it is suitable to state that having a longer deferral time for the investment would lead to better results. There exists a higher revenue potential in the future for such investments due to day-by-day decreasing initial investment costs and the increase in the electricity tariff and distribution prices. The generated model is based on the discounted cash flow calculations, so as the discount rates increase generated loss due the time value of the money would result in decrease in the value with the ROV method. Sensitivity analysis results verify the generated model, since both the discount rates " i " and " r " increase, the value obtained by the ROV method decreases. The experts verified the sensitivity analysis results and the generated model, and noted that solar PV investments offer additional benefits (revenues) that have not been currently monetizable but will become monetizable in the future with the introduction of new regulations, such as carbon taxes. If the investor accepts a lower profit by choosing a lower " i " value, the probability of avoiding losses from the solar PV investment increases. Furthermore, with Turkey's net-zero carbon goals, the solar PV investment may avoid future carbon taxes.

From the investors' point of view, the results have proven that the NPV method may lead to early rejections of the investment projects which may further overlook the potential of the residential PV investments in Turkey. The suggested ROV method may prevent loss of opportunities due to underestimation of the investments by the NPV method. Even though the current status of the residential PV investments in Turkey has not been favorable, rather than a 1730.04 USD loss, there is possibility of making a profit of 2242.87 USD with this particular investment. Thus, the misleading with the NPV method should not prevent the projects at once and the rooftops should not be used for alternative investments that may set back the application of the PV systems in the future. Due to the current trends in PV systems, with respect to the sensitivity analysis results, it is suitable to say that the residential PV

investments will generate better income as an investment in the near future. Therefore, an investor from Turkey should not restrict himself in strict time frame and should allow a longer deferral option in order to achieve higher incomes. The results also highlight that investors should be sensitive for the choice of the discount rates, especially the discount rate determined by the opportunity cost of capital, in order to achieve accurate results since they have a considerable impact on the cash flows generated.

Although there are different studies in literature considering various types and scales of solar energy investments utilizing the ROV method considering different types of options (such as Penizzotto et al. [14], Biancardi et al. [32], and Zhang et al. [42]); all of these studies resulted in favor of the application of ROV for solar energy investments. However, diverse approaches and variations in options studied cause a challenge in comparing results of ROV analysis for PV panel investments. Studies possess differences in terms of scale and underlying factors such as economic and geographical conditions. Therefore, numerical comparison becomes unfeasible. The absence of historical data in emerging markets, such as in Turkey, poses another challenge in terms of validation of numerical findings. Due to these concerns for validation, theoretical comparison of the study has been made through sensitivity analysis for operational functionality of the model. As a result, the method utilized in this study, in line with previous research studies, has been found to be useful for mimicking the effects of managerial flexibility during investment decisions and demonstrating the impact of government incentives on feasibility of investments.

The results of the case study might be beneficial for the Turkish government as well. Undervaluation of the investments carries the risk of labeling the PV systems infeasible and its effects may reach to the public level. So, the future economic advantages, would there be any, could not be foreseen and aims to decrease the carbon emissions might be hardly fulfilled. The substantiated findings of the study indicate that introduction of the ROV method may unveil the hidden potential of such investments and may contribute to the government's objective of reducing carbon emissions. In line with this, major policy changes may lead to an additional value for the potential investors. Future budgets may be planned with additional incentives to enhance the investments since the NPV values of these investments have still been negative. In addition to that, policy decisions may address the future uncertainties in favor of the public. By making these decisions, the advantage with the ROV method might be fostered since it would lead to better results and increase the appetite of the investors.

Two incentive offers were introduced to the ROV model aimed at aiding the Turkish government in promoting the expansion of residential PV investments.

(1) The first offer was a bank loan with a 1.12% interest rate and 300-month credit period to enable distribution of the cost burden along the investment period and make the investors repay the credit with the revenues they would earn. In this scenario based on the expected value of NPV_{trd} , it is observed that the interest rate applied to the bank loan leads the investor to pay a higher amount in return for the loan taken, which eventually affects the NPV_{trd} calculation. When compared with the expected value of NPV_{trd} at the case study, the attractiveness of such investments using a bank loan without a deferral option considerably decreases. However, the ROV results of the case study example turned out to be 14.17 USD, which was almost zero. This means that the decision making for this investment with using a bank loan and having a deferral option of 7 years to make the investment results in a neutral cash flow, which means there is a possibility of making this investment without any loss or income. It can be stated that the utmost probable scenario for the investment with the deferral option would not generate any income. Thus, from the perspective of an investor, such loan may not enhance these investments and the expected values from the NPV calculations and comments on the offered loan may fall short. However, investments on renewable energy production have a hidden value which is generated from the positive impacts on environmental concerns by clean energy production. Since the resulting values are almost zero, which means the investment will not result in any profit or loss, investors from non-profit organizations or the public institutions might still consider to benefit from such a loan. This scenario reveals that a loan incentive for solar PV investments may not lead to higher profits whereas it may lead to the removal of the initial investment cost burden. So, the parties that are not seeking profit can use this incentive in order to construct their PV systems.

(2) The second offer was giving a cash rebate that was equal to the tax (18%) of the initial investment cost. The results of both the ROV method and the NPV method had considerably increased when compared with the results of the case study without the cash rebate. This shows that a cash rebate which will led to a decrease in the initial investment cost can cause the investment to become attractive even today. Lacking a deferral option, the most probable scenario will lead to a 402.33 USD loss in today's money. When the option value generated from the deferral option is added, the most probable scenario will generate a 4116.27 USD income. By just eliminating a small share of the initial investment cost, residential PV investments in Turkey ended up as advantageous investments that generate considerable income and value almost now and in the future. The results justify the severity of the capital cost burden and the requirement of such incentives that decrease the initial investment cost. In line with this, the same effect can be obtained with an increase in the revenues as well and government can also try to increase the revenue generations of such PV systems. However, from an investor's point of view, the initial investment cost still remains as the huge obstacle for the future of these investments. Even with a cash rebate, only the

18% of the initial investment cost can be compensated so the investors still need a considerable amount of saving to make the investment.

As a result of the two incentive scenarios, it can be concluded that more incentives should be provided in order to encourage residential PV investments in Turkey.

7. Conclusion

The residential PV investments are irreversible investments that require considerable initial investment costs and possess uncertain future cash flows due to the inconsistent market conditions. This study proposes the adoption of a relatively new investment valuation method called Real Options Valuation for the Turkish case to overcome the challenges and ease decision-making process for the investors to prevent such early rejections of investments with high ecological value for the Earth's future. The model includes strategic value of flexibility for the investment time, namely the deferral option, in order to make a reliable assessment of PV investments and to maximize the revenue of PV systems. The suggested model handles the uncertainties related to the cost and revenue items of the investments by modelling their future values using stochastic simulations (i.e., GBM). In this context, the model accommodates uncertainties associated primarily with the initial investment cost, stemming from fluctuations in PV panel and inverter prices, as well as the future cash flows influenced by the electricity tariff prices. Thus, the suggested model addresses the identified requirements of the residential PV investments in Turkey.

A case study was conducted for utilization of the model in a real investment decision in Kocaeli, Turkey. The results have proven that while the traditional valuation method, NPV would lead to the rejection of the investment, the proposed ROV method suggests that the option to defer the investment has a considerable value. Thus, results indicate that consideration of uncertainties and incorporating strategic options have a major role in the investment decision. This method ensures that the actual potential of the residential PV investments in Turkey owns has not been underestimated. As uncertainties unfold favorably for investors in the future, the investment would provide profit. Since the residential PV investments also have other benefits for the environment such as decreasing the carbon emissions and providing clean energy, it is of great importance to prevent the early rejection of such investments with this method. Moreover, the NPV method can also mislead the policy-makers in promoting such kind of sustainable investments, which may further hamper the probable potential of these systems.

For the investors, one of the main obstacles for PV systems was identified as the high initial investment cost. Two different scenarios were handled based on incentive examples in alignment with the past and ongoing government incentives around the world to investigate possible policy implications. Results showed that any sort of decrease in the initial investment cost can make residential PV investments in Turkey favorable for the various investors. Particularly, with the low-interest loan incentive, investors have the opportunity to avoid considerable initial investment costs since the financial burden is carried by the bank. This approach allows investors to repay the loan over the lifetime of the residential solar PV system, decreasing the need for significant upfront capital. While it should be noted that there may be certain costs associated with loan management, the overall benefit of reduced initial investment burden can still be advantageous for investors adopting environmentally friendly systems without incurring immediate losses.

As for the research studies, this study provides several contributions. The applicability of the ROV method for residential PV investments in Turkey has been justified and the method has also proven to be useful when evaluating new government incentives that aim to enhance such investments, which can be tested for other countries. Findings of the study contribute to the literature by depicting the status of the residential PV investments through a case example and also by justifying the potential they possess. Additionally, the model can be applied to any sort of investment which mimics the characteristics of residential PV investments in terms of existence of the future uncertainties and the requirement of time flexibility for the investment.

Although the study makes a practical contribution that investors can use the proposed method for the evaluation of investment options, it has to be noted that the proposed method is based on some assumptions regarding the economic conditions and investment incentives which may not hold true for all types of investment alternatives to be carried out in different economic environments. Some of the cost items are taken from the global market prices due to the lack of information in the Turkish solar energy market which can be stated as a potential source of error. The model produced for the case project should not be considered as a generic model. The model focuses on the optimal scenario from an investor's point of view which eventually disregards other possible scenarios that may not end up in the favor of the investor. Future studies are needed to test the viability of ROV for an effective investment evaluation method under different contexts and check the vulnerability of assumptions.

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Data availability

Data will be made available on request.

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