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# Testing the Efficiency of Emerging Markets: Evidence from Nonlinear Panel Unit Tests

**Summary:** In this study, we investigate market efficiency considering nonlinearities by testing the weak-form market efficiency of the stock markets of Brazil, China, Russia, Turkey, and South Africa using recently proposed nonlinear panel unit root tests. The stock markets of these emerging countries are deliberately selected for their market capitalization to form a homogenous panel. The results of nonlinear models indicate that the stock market indexes are stationary and weak-form inefficient. This finding contributes to the contradictory results of the prior research using linear and nonlinear models about the efficiency of emerging stock markets in favor of nonlinear ones. Furthermore, we propose that studies using financial variables consider such nonlinearity in order to achieve more accuracy in findings related to such studies.

**Keywords:** Market efficiency, Stock market, Weak-form efficiency, ESTAR models.

**JEL:** C22.

Efficient market hypothesis (EMH) proposes that security prices always and fully reflect all available information (Eugene Fama 1970, p. 383). If current prices reflect all information, then any price change should result from new information to which the market reacts. This presumption also suggests that investors can achieve no gain by speculation as prices reflect all available information. From this definition, it is also inferred that as the release of new information is unknown and unpredictable in terms of its timing and nature for market participants, the price changes should also be unpredictable and random as a result. Several studies have addressed the randomness of stock prices and have developed the random walk model, which proposes that a unit root represents the unpredictability of stock returns in the stock prices.

Many financial economists around the world have evaluated different stock markets in terms of their efficiency. However, the focus of relevant research has shifted towards emerging markets, referring to widespread international investments in those assets. Emerging economies are generally defined based on a certain level of GDP *per capita*, in addition to their common characteristics of high volatility, higher-than-average return, and less mature capital markets. From an investor's perspective, less efficient markets are preferable as there possibly exist abnormally high returns. This inference paves the way for researchers to test the efficiency of stock markets of emerging countries. Since the number of such studies is limited, there is no consensus about emerging stock markets' efficiency.

Variance ratio tests, run tests, and linear unit root tests have been the conventional methods used in testing the efficiency of stock markets. However, the nonlinearities in the stock markets' price movements call for such nonlinear models to be conducted at best in the analyses. It is accepted that nonlinearity in stock market data stems from market frictions and transaction costs. Transaction costs, which are the spread between ask and bid prices, the arbitrage limits representing the allowance of short-selling, and borrowing constraints make arbitrage less profitable in small deviations from the equilibrium, this means the size of the deviation matters for the profitability of the arbitrage, as well as for reversion to the equilibrium. Other more behavioral factors such as the interaction of heterogeneous agents or heterogeneity of investors' objectives may also create persistent deviation from the equilibrium in the stock prices (Mubariz Hasanov and Tolga Omay 2008).

In this framework, following the study of George Kapetanios, Yongcheol Shin, and Andy Snell (2003), Nuri Ucar and Omay (2009) modeled the panel unit root test procedure in an exponential smooth transition autoregressive (ESTAR) framework. The model proposed by Ucar and Omay (2009) has been used to test financial market efficiency and analysis, including nonlinear data generation such as foreign direct investment, unemployment, and others. Some other comprehensive models have been proposed to better address nonlinearities in economic variables, such as the models of Furkan Emirmahmutoğlu and Omay (2014), Ayşegül Çorakçı, Emirmahmutoğlu, and Omay (2017), Omay, Çorakçı, and Emirmahmutoğlu (2017), Omay, Hasanov, and Shin (2018) and Omay, Muhammad Shahbaz, and Chris Steward (2021). These models capture state-dependent nonlinearity in TAR form and AESTAR form, simultaneous effects of structural break and TAR nonlinearity, and time-dependent nonlinearity, respectively.

## 1. Literature Survey

When evaluating the predictability of stock market returns in the framework of market efficiency, many researchers have used nonlinear models under different assumptions, alone or together with linear models for comparison purposes. Prior research as particularly considered nonlinear dynamics within the US and the UK stock market returns, such as the studies of Mark Leung, Hazem Daouk, and An-Sing Chen (2000), Dick van Dijk and Philip Hans Franses (2000) and Philip Shively (2003). David G. McMillan (2005) analyzed the stock index returns of France, Germany, Japan, Singapore, Hong Kong, and Malaysia using linear AR and nonlinear QLSTAR models. He reported that the nonlinear model outperforms the linear model in forecasting market behaviors. Paresh Narayan (2005) investigated the stock prices in the equity markets of Australia and New Zealand, and used an unrestricted two-regime threshold autoregressive model. In that research, it is indicated that the stock prices of both countries are generated by nonlinear processes that are characterized by a unit root, consistent with the EMH.

Sei-Wan Kim, André V. Mollick, and Kiseok Nam (2008) examined common nonlinearities in long-horizon stock returns of G-7 countries by using both LSTAR and ESTAR models, and they found that nonlinear models clearly outperform linear models in in-sample and most out-of-sample forecasting attempts. Hasanov and Omay

(2008) also used both the autoregressive model and STAR model when analyzing out-of-sample forecasting of monthly returns of the Athens and Istanbul stock exchanges. They concluded that the STAR model outperformed the other.

Kian-Ping Lim (2007) demonstrated that the nonlinear dependence in stock returns is relatively localized in time and suggested that market efficiency evolves over-time. It was also found that the US equity market is the most efficient, while the Argentine stock market is at the bottom of the ranking.

Qaiser Munir and Kasim Mansur (2009) investigated the behavior of the Kuala Lumpur Stock Exchange Composite Index (KLSCI) by using a two-regime threshold autoregressive (TAR) model developed by Mehmet Caner and Bruce E. Hansen (2001). The results indicated that the KLSCI is a nonlinear series characterized by a unit root process consistent with the EMH.

Onur Gozbası, İlhan Kucukkapan, and Saban Nazlıoğlu (2014) used the nonlinear ESTAR unit root test then recently developed by Robinson Kruse (2011) to analyze Borsa Istanbul's efficiency. Their results showed that the Turkish stock market has weak-form efficiency. The research focus of Seher Nur Sülkü and Emre Ürkmez (2018) examined the nonlinear dynamics of the Borsa Istanbul index daily returns with a BDS nonlinearity test. They found that daily returns can be forecast in short-term periods, but are not predictable in the long-run.

Aycan Hepsag and Burçay Yasar Akcalı (2015) investigated the weak-form efficiency of the stock markets of G-7 and E-7 countries. They tested whether the stock market indices follow the random walk model or not using an asymmetric nonlinear unit root test. The empirical findings showed that the stock markets of France, Italy, Japan, and the USA have weak-form efficiency, but the stock markets of Canada, Germany, and the UK are not as such. The empirical results also showed that Brazil, China, India, Indonesia, Mexico, and Turkey have weak-form efficiency, but the stock market of Russia is not so.

Fuzuli Aliyev (2019) examined the market efficiency of Borsa Istanbul by using nonlinear ARCH and STAR models and also a linear AR model and random walk model. By comparing the forecast performance powers, he concluded that the STAR model outperforms random walk; that is, Borsa Istanbul returns are predictable in the given period. Contrary to linear-level studies, these findings show that Borsa Istanbul is not weak-form efficient at the nonlinear level.

Ceyda Aktan, Perihan İren, and Omay (2019) tested the weak-form market efficiency of 32 European stock markets by categorizing them within three different groups: frontier, emerging, and developed. The results showed a meaningful relationship between varying levels of economic development and weak-form market efficiency. They concluded that considering the nonlinear structure of the stock market indices, linear models might lead to the wrong conclusions regarding market efficiency.

These models mostly used nonlinear time-series unit root tests; however, our contribution in this study is to consider market spillovers and other common factors by using the panel data structure. Using the panel approach, the misspecifications are limited in modeling the correct specification of the stock markets. In time-series analyses, there is no opportunity to account for global factors such as crisis spillovers or

contamination effects of other countries, which lead to miss-modeled analysis and yield results such as weak-form efficiency. This is the main novelty of the present study by primarily focusing on nonlinearity in stock markets. In this framework, the key motivation is to evaluate the stock market efficiency of emerging market countries, namely Brazil, China, Russia, Turkey, and South Africa using the recently generated nonlinear models to fill the gap in the field of analysis. From among 26 emerging markets, these countries are selected to create a homogenous panel in terms of stock market capitalization as a percentage of GDP. New-generation ESTAR models are used in order to reach a more comprehensive conclusion about the predictability of returns of the popular emerging stock markets. It is thought that the findings of the study would guide international investors in their equity investments, as well as academicians in their studies about the efficiency of the stock markets and the predictability of returns. The rest of the paper is structured as follows: Section 2 introduces the methodologies used in the study, Section 3 is devoted to the data and empirical analysis, and Section 4 concludes.

## 2. Methodology

Before explaining the methodologies used in the study, the approach adopted in this paper is summarized, and the nature of second-generation panel unit root tests and the features of nonlinear panel unit root tests is discussed. For second-generation panel unit root tests, there are two common types of cross-section dependency remedy methods; the first of these methods is inserting the unobservable factor structure into the testing model, and the second is bootstrapping. Examples of the first model structure are M. Hashem Pesaran's (2007) CIPS or CADF method and Jushan Bai and Serena Ng's (2003) PANIC method. In the Pesaran (2007) method, the unobservable factor is eliminated by inserting the average of the dependent and independent variables into the model. In comparison, the factors are estimated in the Bai and Ng (2003) method. Examples for the second type include the studies of Yoosoon Chang (2004), Ucar and Omay (2009) (UO), and Emirmahmutoglu and Omay (2014) (EO).

Following these studies, Omay, Hasanov, and Shin (2018) (OHS) developed a structural break panel unit root test using sieve bootstrap and Pesaran (2007)'s Common Correlated Effect Estimator (CCE) methods. Their study showed that the remedies made with the factor in the structural break models mimic the structural breaks instead of the unobserved factor. This problem makes it inconvenient to use factor variables in panel unit root tests with the data that exhibit time-dependent structural breaks. There are two-panel unit root tests that test the structural break in our study. The first of these is Omay, Shahbaz, and Steward (2021) (OSS), and the second is OHS (2018) tests. With both tests, the stationarity is obtained for the sample under consideration. These results show that the sample is subject to structural breaks. As mentioned above, the OHS (2018) test also has the Pesaran (2007)-type factor remedy version; yet, we do not use the CCE version of the panel unit root tests because the CCE estimator mimics the structural breaks. Finally, UO (2009), EO (2014), Omay, Corakci, and Emirmahmutoglu (2017) (OCE) and OSS (2021) tests eliminate cross-section dependency only with the bootstrap method. Apart from these tests, there are very

few nonlinear panel unit root tests. In this respect, it is essential to apply the Chang (2004) test, which uses the bootstrap method to ensure consistency<sup>1</sup>.

In the model for nonlinear unit root tests of Ucar and Omay (2009),  $z_{it}$  is defined as the panel exponential smooth transition autoregressive process of order one (PESTAR(1)) on the time domain  $t = 1, 2, \dots, T$  for the cross-section units  $i = 1, 2, \dots, N$ . Consider the following PESTAR process with fixed effect parameter  $\alpha_i$  generating  $z_{it}$  by Equation (1):

$$\Delta z_{i,t} = \alpha_i + \phi z_{i,t-1} + \gamma_i z_{i,t-1} [1 - \exp(-\theta_i z_{i,t-d}^2)] + \varepsilon_{i,t}. \quad (1)$$

In this formula,  $d \geq 1$  is the delay parameter and  $\theta_i \geq 0$  represents the speed of revision for all units;  $\varepsilon_{i,t}$  is a serially and cross-sectionally uncorrelated disturbance term with zero mean and variance  $\alpha_i^2$ .

Following the previous literature, Ucar and Omay (2009) set  $\Phi_i = 0$  for all  $i$  and  $d = 1$ , which gives the specific PESTAR(1) model:

$$\Delta z_{i,t} = \alpha_i + \gamma z_{i,t-1} + \gamma_i z_{i,t-1} [1 - \exp(-\theta_i z_{i,t-d}^2)] + \varepsilon_{i,t}. \quad (2)$$

The nonlinear panel data unit root test based on regression (1) with augmented lag variables is modeled for testing the null hypothesis  $\theta_i$  for all  $i$  against  $\theta_i \geq 0$  for some  $i$ . However, as the direct testing of the null hypothesis is problematic since  $\gamma_i$  is not identified under the null hypothesis, first-order Taylor series expansion to the PESTAR(1) model around  $\theta_i = 0$  for all  $i$  are calculated. Hence, the auxiliary regression is modified as:

$$\Delta z_{i,t} = \alpha_i + \delta_i z_{i,t-1}^3 + \varepsilon_{i,t}. \quad (3)$$

Here,  $\delta_i = \theta_i \gamma_i$ . The hypotheses for unit testing based on Equation (2) are:

$$H_0 : \delta_i = 0 \text{ for all } i \quad (\text{linear nonstationary})$$

$$H_0 : \delta_i = 0 \text{ for all } i \quad (\text{nonlinear stationary})$$

By using the nonlinear time series framework developed by Kapetanios, Shin, and Snell (2003), or KSS hereafter, the panel unit root tests are computed by taking the average of individual KSS statistics. The KSS test for the  $i^{\text{th}}$  item is the  $t$ -statistics for testing  $\delta_i = 0$  in Equation (4):

$$t_{i,NL} = \frac{\Delta z_i' M_t z_{i,-1}^3}{\alpha_{i,NL} (z_{i,-1}' M_t z_{i,-1})^{3/2}}. \quad (4)$$

$\hat{\alpha}_{i,NL}^2$  is a consistent estimator such that  $\hat{\alpha}_{i,NL}^2 = \frac{\Delta z_i' M_t z_i}{(T-1)}$  where  $M_t = I_T - \tau_T (\tau_T' \tau_T)^{-1} \tau_T'$  with:

$$\Delta z_i = (\Delta z_{i-1}, \Delta z_{i-2}, \dots, \Delta z_1)' \text{ and } \tau_T = (1, 1, \dots, 1).$$

<sup>1</sup> We did not include the linear panel unit root test explanation, which we will call Chang (2004), or the bootstrap IPS test, as it has a slight difference from the UO (2009) test. If the cubed term in the UO (2009) test is taken from the first order, the expression of the Chang (2004) test is obtained.

Assuming that all the statistics  $t_{i,NL}$  are random variables with finite means and variances, then average statistics  $t_{i,NL}$  have the limiting standard normal distribution as  $N \rightarrow \infty$  such that:

$$\bar{Z}_{NL} = \frac{\sqrt{N}(\bar{t}_{NL} - E(t_{i,NL}))}{\sqrt{\text{var}(t_{i,NL})}} \tag{5}$$

In the above formulation,  $E_i$  and  $\text{var}(t_{i,NL})$  can be found in Table 1 of Ucar and Omay (2009) and  $\bar{t}_{NL} = N^{-1} \sum_{t=1}^N t_{NL}$ .

It is known that for many of the panel studies, cross-section dependency may arise for several reasons, such as spatial correlations, spillover effects, economic distance, omitted global variables, or common unobserved shocks. It is accepted that cross-section dependency can cause biased estimates and misleading inferences for smaller as well as larger panels. Pesaran (2004) CD test or Omay and Elif O. Kan (2010) nonlinear CD test, which performs an additional cross-sectional dependency test, were not used in our study since the Bootstrap method is used to effectively eliminate these problems whether there is a cross-section dependency or not. These cross-section dependency test results are necessary preliminary tests for CCE or factor type estimation methods. If there is no cross-section dependency, the Bootstrap method will act according to the existing distribution and determine the actual distribution in this way.

In order to address symmetric or asymmetric ESTAR nonlinearity as the alternative for the heterogeneous panels, Emirmahmutoğlu and Omay (2014), or EO hereafter, combined the nonlinear panel unit root test together with the framework of Robert Sollis (2009). The results show that:

$$\Delta y_{it} = G_{it}(\gamma_{1i}, y_{i,t-1}) \left\{ S_{it}(\gamma_{2i}, y_{i,t-1}) \rho_{1i} + \left( 1 - S_{it}(\gamma_{2i}, y_{i,t-1}) \right) \rho_{2i} \right\} y_{i,t-1} + \varepsilon_{it}; \tag{6}$$

$$G_{it}(\gamma_{1i}, y_{i,t-1}) = 1 - \exp(-\gamma_{1i} y_{i,t-1}^2), \gamma_{1i} \geq 0 \text{ (for all } i); \tag{7}$$

$$S_{it}(\gamma_{2i}, y_{i,t-1}) = [1 + \exp(-\gamma_{2i} y_{i,t-1})]^{-1}, \gamma_{2i} \geq 0 \text{ for all } i. \tag{8}$$

By the use of the methodology of Sollis (2009), a nonlinear asymmetric heterogeneous panel is generated. The ESTAR transition process is realized between the central and the outer regime depending on the state variable  $y_{i,t-1}$ . If the conditions  $\gamma_{1i} > 0$  and  $\gamma_{1i} \rightarrow \infty$  exist, then the deviation of  $y_{i,t-1}$  will be great enough for the transition to take place. In the case that the transition function takes the values 0 and 1, the outer regime will become the following:

- if the change is in the negative direction of the state variable,  $\Delta y_{it} = \rho_{i2} y_{i,t-1} + \varepsilon_{it}$ ;
- if the change is in the positive direction of the state variable,  $\Delta y_{it} = \rho_{i1} y_{i,t-1} + \varepsilon_{it}$ .

The EO test proposes that in cases where the condition  $\rho_{i1} \neq \rho_{i2}$  holds, an asymmetric autoregressive adjustment exists. However, in cases where  $\rho_{i1} = \rho_{i2} = \rho_i$ , then a symmetric ESTAR specification holds. The model was modified as follows in order to eliminate the possibility of the serial correlation of the errors:

$$\Delta y_{it} = G_{it}(y_{1i}, y_{i,t-1}) \left\{ S_{it}(y_{2i}, y_{i,t-1}) \rho_{1i} + (1 - S_{it}(y_{2i}, y_{i,t-1})) \rho_{2i} \right\} y_{i,t-1} + \sum_{j=1}^{\rho_i} \delta_{ij} \Delta y_{i,t-j} + \varepsilon_{it}. \quad (9)$$

There exist some unidentifiable parameters, such as  $y_{2i}$ ,  $\rho_{1i}$ , and  $\rho_{2i}$ , and this problem is overcome by Taylor approximation, the result of which is:

$$\Delta y_{it} = \phi_{1i} y_{i,t-1}^3 + \phi_{2i} y_{i,t-1}^4 + \varepsilon_{it}. \quad (10)$$

After augmentation, the equation becomes the following:

$$\Delta y_{it} = \phi_{1i} y_{i,t-1}^3 + \phi_{2i} y_{i,t-1}^4 + \sum_{j=1}^{\rho_i} \delta_{ij} \Delta y_{i,t-j} + \varepsilon_{it}. \quad (11)$$

The null hypothesis is stated as  $H_0 : \phi_1 = \phi_2 = 0$ , and when rejected, then the null hypothesis of symmetric ESTAR nonlinearity can be tested against the alternative hypothesis of asymmetric ESTAR nonlinearity. The cross-section dependency is addressed by the sieve bootstrap method.

The heterogeneous panel unit root test of Walter Enders and Clive W. J. Granger (1998) was improved by Çorakç1, Emirmahmutoglu, and Omay (2017), or CEO hereafter, who used the model of Ucar and Omay (2009) while applying the sieve bootstrap method to eliminate the cross-section dependency problem. CEO utilized the following PESTAR(1) process as proposed by Ucar and Omay (2009) for determining the panel asymmetric TAR (PTAR) model, while Çorakç1, Emirmahmutoglu, and Omay (2017) utilized the PESTAR(1) process as proposed by Ucar and Omay (2009):

$$\Delta y_{it} = \alpha_i + \phi_i y_{i,t-1} + \gamma_i y_{i,t-1} [1 - \exp(-\theta_i y_{i,t-d}^2)] + \varepsilon_{it}. \quad (12)$$

Here,  $\alpha_i$  is the fixed effect and  $d \geq 1$  represents delay parameters.

The approach of CEO modified the above equation in order to obtain the PTAR(1) process by replacing the indicator function with the transition function ( $G_{it}(s_{it} = y_{i,t-1}; \theta, c_i = 0) = 1 - \exp(-\theta y_{i,t-1}^2)$ ) and specified the following equation:

$$\Delta y_{it} = \alpha_i + \rho_{i1} I_{it} y_{i,t-1} + \rho_{i2} (1 - I_{it}) y_{i,t-1} + \eta_{it}. \quad (13)$$

In this formulation,  $I_{it}$  will be 1 if  $y_{i,t-1} \geq 0$  and will be 0 if  $y_{i,t-1} < 0$ . In the model,  $\eta_{it}$  represents a stationary process with zero mean.

A more comprehensive panel unit root test was proposed by Omay, Çorakç1, and Emirmahmutoglu (2017), or OCE hereafter, incorporating the works of both Kyung So Im, Pesaran, and Yongcheol Shin (2003) and Sollis (2009). Similar to the models of EO and CEO, this model also uses the sieve bootstrap method to eliminate the cross-sectional dependency problem. OCE proposed  $y_{it}$  to be a panel ST-TAR process that has a changing trend and will be generated from one of the following three smooth transition (ST) processes:

$$\text{Model A: } y_{it} = \alpha_{i1} + \alpha_{i2} S_{it}(y_i, \tau_i) + \varepsilon_{it}. \quad (14)$$

$$\text{Model B: } y_{it} = \alpha_{i1} + \beta_{i1} t + \alpha_{i2} S_{it}(y_i, \tau_i) + \varepsilon_{it}. \quad (15)$$

$$\text{Model C: } y_{it} = \alpha_{i1} + \beta_{i1}t + \alpha_{i2}S_{it}(\gamma_i, \tau_i) + \beta_{i2}tS_{it}(\gamma_i, \tau_i) + \varepsilon_{it}. \tag{16}$$

In this test,  $S_{it}(\gamma_i, \tau_i)$  represents the logistic smooth transition function that has sample size  $T$  and  $N$  units. It is shown as:

$$S_{it}(\gamma_i, \tau_i) = [1 + \exp\{-\gamma_i(t - \tau_i T)\}]^{-1}, \gamma_i > 0. \tag{17}$$

$\varepsilon_{it}$  is generated using the following:

$$\Delta\varepsilon_{it} = \rho_{i1}I_{it}\varepsilon_{i,t-1} + \rho_{i2}(1 - I_{it})\varepsilon_{i,t-1} + \eta_{it}. \tag{18}$$

TAR model is applied where:

$$I_{it} = \begin{cases} \text{if } 1, \text{ then } y_{t-1} \geq 0 \\ \text{if } 0, \text{ then } y_{t-1} < 0 \end{cases}$$

The null hypothesis that there exists a unit root is accepted when  $\rho_{i1} = \rho_{i2} = 0$  (for all  $i$ ) against the alternative of a stationary panel ST-TAR process with symmetric adjustment when  $\rho_{i1} = \rho_{i2} < 0$  (for some  $i$ ). In this model, the proposed transition between regimes is realized through a smooth transition and the smoothness is expressed using the  $\gamma_i$  parameter. The transition function  $S_{it}(\gamma_i, \tau_i)$  is said to have extreme values of 0 and 1. The three models above are used for testing the following hypotheses:

Null hypothesis (H0)  $y_{it} = \mu_{it}, \mu_{it} = \mu_{i,t-1} + \varepsilon_{it}, \mu_{i0} = \psi_i$

Alternative hypothesis (H1) Model A, Model B, or Model C

or

Null hypothesis (H0)  $y_{it} = \mu_{it}, \mu_{it} = \kappa_i + \mu_{i,t-1} + \varepsilon_{it}, \mu_{i0} = \psi_i$

Alternative hypothesis (H1) Model B or Model C

In order to calculate the test statistics, first, the deterministic components for every cross-sectional unit are estimated, and then in the second phase, then linear least squares residuals are obtained from the below alternatives:

Model A:  $\hat{\varepsilon}_{it} = y_{it} - \hat{\alpha}_{i1} - \hat{\alpha}_{i2}S_{it}(\hat{\gamma}_i, \hat{\tau}_i);$

Model B:  $\hat{\varepsilon}_{it} = y_{it} - \hat{\alpha}_{i1} - \hat{\beta}_{i1}t - \hat{\alpha}_{i2}S_{it}(\hat{\gamma}_i, \hat{\tau}_i);$

Model C:  $\hat{\varepsilon}_{it} = y_{it} - \hat{\alpha}_{i1} - \hat{\beta}_{i1}t - \hat{\alpha}_{i2}S_{it}(\hat{\gamma}_i, \hat{\tau}_i) - \hat{\beta}_{i2}tS_{it}(\hat{\gamma}_i, \hat{\tau}_i).$

However, in the second step, the following model is used to test for unit roots:

$$\Delta\hat{\varepsilon}_{it} = \rho_{i1}I_{it}\hat{\varepsilon}_{i,t-1} + \rho_{i2}(1 - I_{it})\hat{\varepsilon}_{i,t-1} + \sum_{j=1}^{k_i} \delta_{ij}\Delta\hat{\varepsilon}_{i,t-j} + \eta_{it}. \tag{19}$$

The  $t$ -statistic used can be shown as:

$$t_{ij} = \frac{\sqrt{T-k_i-2}(\hat{\varepsilon}'_{ij,-1}M_{Q_{ij}}\Delta\hat{\varepsilon}_i)}{(\hat{\varepsilon}'_{ij,-1}M_{Q_{ij}}\hat{\varepsilon}_{ij,-1})^{\frac{1}{2}}(\Delta\hat{\varepsilon}'_iM_{X_{ij}}\Delta\hat{\varepsilon}_i)^{\frac{1}{2}}} j = 1,2. \tag{20}$$

The  $F$ -statistic to test the null hypothesis is defined as:

$$F_i = (R\hat{P}_i)'[\hat{\sigma}_i^2R(C'_iC_i)^{-1}R']^{-1}(R\hat{P}_i)/2. \tag{21}$$

Omay, Hasanov, and Shin (2017), or OHS hereafter, addressed the problems related to previous unit root testing approaches, which mainly involve the assumption of structural breaks occurring concurrently and the presence of correlation errors in cross-sectional areas. It is a fact that both of these invalidate traditional panel unit root and cointegration tests. Therefore, to improve the current dynamic panel unit root tests, OHS developed a panel unit root test characterized by both slowly moving trends (SMT) and cross-section dependence (CSD). In this new approach:

Let  $y_{i,t}$  be:

$$y_{it} = \phi_{it} + x_{it}. \quad (22)$$

$$x_{it} = \mu_i x_{it-1} + \varepsilon_{it}. \quad (23)$$

Here,  $\phi_{it}$  is the slow-moving deterministic trend and  $\varepsilon_{it}$  is the error term. The hypotheses of the test are:

$$H_0: \rho_i = 0$$

$$H_1: \rho_i < 0$$

Here,  $\rho_i = -(1 - \mu_i)$ .

$\phi_{it}$  is modeled in the study using the logistic STR:

$$\text{Model A: } \phi_{it} = \alpha_{1i} + \alpha_{2i} S_{it}(\gamma_i, \tau_i);$$

$$\text{Model B: } \phi_{it} = \alpha_{1i} + \beta_{1i} t + \alpha_{2i} S_{it}(\gamma_i, \tau_i);$$

$$\text{Model C: } \phi_{it} = \alpha_{1i} + \beta_{1i} t + \alpha_{2i} S_{it}(\gamma_i, \tau_i) + \beta_{2i} t S_{it}(\gamma_i, \tau_i).$$

In these models  $S_{it}(\gamma_i, \tau_i)$  represents the individual-specific logistic smooth transition functions and controls the transition between two regimes. It has extreme values of 0 and 1 and is shown as:

$$S_{it}(\gamma_i, \tau_i) = [1 + \exp\{\gamma_i(t - \tau_i T)\}]^{-1}, \quad \gamma_i > 0. \quad (24)$$

The unit root test statistics are calculated in two steps. First, the non-linear least squares algorithm is used, the deterministic components of the model are calculated, and the residuals are collected:

$$\text{Model A: } \hat{x}_{it} = y_{it} - \hat{\alpha}_{1i} - \hat{\alpha}_{2i} S_{it}(\hat{\gamma}_i, \hat{\tau}_i);$$

$$\text{Model B: } \hat{x}_{it} = y_{it} - \hat{\alpha}_{1i} - \hat{\beta}_{1i} t - \hat{\alpha}_{2i} S_{it}(\hat{\gamma}_i, \hat{\tau}_i);$$

$$\text{Model C: } \hat{x}_{it} = y_{it} - \hat{\alpha}_{1i} - \hat{\beta}_{1i} t - \hat{\alpha}_{2i} S_{it}(\hat{\gamma}_i, \hat{\tau}_i) - \hat{\beta}_{2i} t S_{it}(\hat{\gamma}_i, \hat{\tau}_i).$$

In the second step, the regression  $\Delta \hat{x}_{it} = \rho_i \hat{x}_{i,t-1} + \eta_{it}$  is used to calculate the ADF  $t$ -statistic, where  $\rho_i = 0$ . The  $t$ -statistics can be calculated as follows:

$$t_i = \frac{\Delta x'_{it} M_T x_{i-1}}{\hat{\sigma}_i (x'_{i-1} M_T x_{i-1})^{1/2}}. \quad (25)$$

These test statistics are derived with the assumption that there are no cross-sectional dependences in errors, and OHS mentioned that the cross-sectional dependency needs to be allowed. For the CCE-based unit root test statistics, the following heterogeneous panel regression is expressed as the starting point:

$$\Delta \hat{x}_{i,t} = \rho_i \hat{x}_{i,t-1} + \varepsilon_{i,t}. \quad (26)$$

$$\begin{aligned} \varepsilon_{it} &= \lambda_i f_t + u_{it}, \\ u_{i,t} &\sim \text{iid. } N(0, \sigma_u^2) \text{ and } f_t \sim N(0, \sigma_f^2). \end{aligned} \tag{27}$$

In this regression,  $f_t$  represents the unobserved common factor, which is approximated by the cross-section averages of  $\Delta \hat{x}_{i,t}$  and  $\hat{x}_{i,t-1}$ . It is shown as:

$$\Delta \hat{x}_{it} = \rho_i \hat{x}_{i,t-1} + \varphi_i \bar{\hat{x}}_{t-1} + \zeta_i \Delta \bar{\hat{x}}_t + e_{it}. \tag{28}$$

This then becomes:

$$\Delta \hat{x}_{it} = \rho_i \hat{x}_{i,t-1} + \varphi_i \bar{\hat{x}}_{t-1} + \sum_{j=0}^p \theta_{ij} \Delta \bar{\hat{x}}_{t-j} + \sum_{j=1}^p \delta_{i,j} \Delta \hat{x}_{i,t-j} + e_{it}. \tag{29}$$

Although the OHS test has both a bootstrap and a CADF version, we used the Bootstrap version in the study to be consistent with other tests which they have only bootstrap versions.

In the last model of the FIPS panel unit root test developed by Omay, Shahbaz, and Steward (2021), the panel autoregressive process, which is denoted by  $u_{it}$ , is estimated by using the following regression<sup>2</sup>:

$$u_{it} = \alpha_i + d_i(t) + \phi_{i,1} u_{i,t-1} + \lambda_i t + \varepsilon_{it}. \tag{30}$$

In the above equation,  $\varepsilon_{it}$  is the error term with variance  $\sigma_i$  and  $d_i(t)$  is the deterministic function of  $t$ . In order to solve the problem in cases where the structure of  $d_i(t)$  is unknown, as testing  $\phi = 1$  is burdensome, Omay and Iren (forthcoming) proposed to use the FIPS test while incorporating Fourier expansion as stipulated below to estimate  $d_i(t)$ :

$$d_i(t) = \alpha_{i,0} + \alpha_{i,k} \sin\left(\frac{2\pi k_i t}{T}\right) + \beta_{i,k} \cos\left(\frac{2\pi k_i t}{T}\right). \tag{31}$$

In the above equation,  $k_i$  and  $T$  denotes the frequency and the observation number, respectively. The frequency of 1 is expressed as a good estimation to a model representing the structural change. The testing regression is specified as:

$$u_{it} = \alpha_i + d_i(t) + \phi_{i,1} u_{i,t-1} + \lambda_i t + \varepsilon_{it}. \tag{32}$$

The test is realized by taking the average of the individual Ender and Lee (2012) (EL) statistics across the entire panel and the t-statistic expressed by the following when  $\rho_i = 0$ :

$$t_{i,F} = \frac{\Delta u_i' M_i u_{i,-1}}{\hat{\sigma}_{i,sl} (u_{i,-1}' M_i u_{i,-1})^{1/2}}, \tag{33}$$

$\hat{\sigma}_{i,t}$  which is the consistent estimator, is calculated by:

$$\hat{\sigma}_{i,sl}^2 = \Delta y_i' M_i \Delta y_i / (T - 1), \quad M_i = I_T - \tau_T (\tau_T' \tau_T)^{-1} \tau_T'. \tag{34}$$

<sup>2</sup> The fractional version of this test is newly proposed by Emirmahmutoglu et al. (2021) (EOSN test).

The mean group statistic calculated by using the individual  $t$ -statistics is tested for the unit root:

$$\bar{t}_{FIPS} = N^{-1} \sum_{i=1}^N t_i, F. \quad (35)$$

### 3. Data and Results

In this study, the weekly stock market indexes of Russia, Brazil, China, South Africa, and Turkey covering the period from October 2009 to October 2019 are considered. The weekly data are collected from Thompson Reuters, and these countries form a set of alternatives with similar risk and return estimates from an equity investor's perspective. To build a relatively homogenous panel, we have used the market capitalization of the emerging market countries<sup>3</sup>. Table 1 reports the details of the stock market indexes, and the following models are used to test the existence of a unit root in the dataset indicating that stock prices follow a random walk pattern. The models are UO (2009), EO (2014), CEO (2017), OCE (2017), OHS (2018) and OSS (2018).

**Table 1** Summary Statistics for Stock Market Indexes, October 2009–November 2019

Country	Series	Number of obs.	Mean	Std. dev.
Turkey	Borsa Istanbul 100 index	522	78,265	16,741
Brazil	Ibovespa Brasil Sao Paulo stock exchange index	522	63,895	14,388
China	Shanghai composite	522	2,747,885	6,484,325
Russia	MOEX	522	1,757,998	4,180,631
South Africa	JSE FTSE	522	45,009	10,471

Source: Authors' calculations.

All statistical analyses are performed using the WinRats<sup>4</sup> program. Table 2 presents the results of the unit root test in order to evaluate the weak-form efficiency of the stock markets of Russia, Brazil, China, South Africa, and Turkey, all of those data are constructed as a panel.

Amongst the other second-generation panel unit root tests, the methodology of Chang (2004) is used as he proposes a bootstrap unit root test that, contrary to the previous tests, successfully overcomes the nuisance parameters problem in panels with cross-sectional dependence. It could be stated that only the linear methodology of Chang (2004) can reveal that the emerging stock markets as a group are nonstationary and efficient. The model of Çorakçı, Emirmahmutoglu, and Omay (CEO 2017) applies the sieve bootstrap method to eliminate the cross-section dependency problem and includes the asymmetric panel TAR (PTAR) model. This method also stipulates that those stock markets as a panel are efficient.

<sup>3</sup> The market capitalization as a percentage of GDP is 69% for Brazil (2020), 34.5% (2018) for Russia, 59.6% (2019) for China, 35.3% (2020) for Turkey, and 46.7 (2020) for South Africa. India is removed from the analysis from the popular group of BRICS countries as its market capitalization as a percentage of GDP is 115% (2020) and deviates from the selected countries. Turkey is included in the group.

<sup>4</sup> WIN-RATS (Regression Analysis of Time Series) is an econometrics and time series analysis software package and you can also find the similar test in NONSTAT program.

**Table 2** Results of Unit Root Tests

	Test	Test type	Intercept	Intercept trend	
Linear	Chang (2004)	t-test	-1.804 (0.332)	-2.625 (0.195)	Non-stationary/Efficient
	CEO (2017) PTAR (SD)	F-test	1.945 (0.515)	3.692 (0.255)	Non-stationary/Efficient
State dependent nonlinearity	UO (2009) PSTAR	t-test	-1.919 (0.369)	-2.950* (0.087)	Stationary/Inefficient
	EO (2014) PAESTAR	F-test	3.344 (0.202)	5.726* (0.065)	Stationary/Inefficient
Structural break	OSS(2021) test	t-test	-3.548** (0.011)	-3.884** (0.047)	Stationary/Inefficient
		t-test	Model A	-3.623* (0.060)	Stationary/Inefficient
Structural break	OHS (2018)	t-test	Model B	-4.073* (0.080)	Stationary/Inefficient
		t-test	Model C	-4.580*** (0.000)	Stationary/Inefficient
		F-test	Model A	7.647* (0.080)	Stationary/Inefficient
Structural break + state dependent nonlinearity	OCE (2017)	F-test	Model B	8.899* (0.060)	Stationary/Inefficient
		F-test	Model C	11.602*** (0.000)	Stationary/Inefficient

**Notes:** This table shows the results of the Chang (2004) test; Omay, Corakci, and Emirmahmutoglu (2017) test as (OCE); Ucar and Omay (2009) test as (UO); Emirmahmutoglu and Omay (2014) test as (EO); Omay, Hasanov, and Shin (2018) test as (OHS) and Corakci, Emirmahmutoglu, and Omay (2017) test as (CEO) for the sample. OSS (2021), Omay, Shahbaz, and Steward (2021). The data period is from Oct 2009- November 2019. The applicable critical values of each model are given in the parenthesis.

**Source:** Authors' calculations.

The state-dependent nonlinearity is covered by Ucar and Omay (2009) and Emirmahmutoglu and Omay (2014). These models yield the finding that the emerging stock markets as a panel are stationary and inefficient. The result of the FIPS test of Omay, Shahbaz, and Steward (2021), wherein the structural break is removed with a flexible Fourier function incorporating Fourier expansion, confirms the panel to be stationary and inefficient.

The three models of Omay, Hasanov, and Shin (OHS 2017) show that the panel of Russia, Brazil, China, South Africa, and Turkey is stationary and inefficient.

The panel unit root test proposed by Omay, Çorakçı, and Emirmahmutoglu (OCE 2017) also uses the sieve bootstrap method to eliminate the cross-sectional dependency problem, and the deterministic components are specified for three different models. This model simultaneously allows for the existence of structural breaks. The results of all three models show that the panel of Russia, Brazil, China, South Africa, and Turkey is stationary and inefficient.

These results should be analyzed in conjunction with the most current studies using linear and nonlinear models, or both, regarding the efficiency of the relevant stock markets. The most recent analysis that uses variance ratio tests as the linear model and the BDSL test as the nonlinear model (Siva Kiran and Rao.R Prabhakar 2019) reveals that when using the linear model, the stock markets of Brazil, Russia, China, and South Africa (within the group of BRICS) seem efficient. Still, when using the nonlinear model for all the BRICS markets, the random walk hypothesis is rejected due to the nonlinear dependence. Another relevant study is that of K. G. Suresh, Anto Joseph, and Garima Sisodia (2013), who used the nonlinear panel-unit-root test model of Ucar and Omay (2009) to analyze the efficiency of the stock markets of the BRICS countries (Brazil, Russia, India, China, and South Africa). They reported that these emerging stock indices have a nonlinear data-generating process and are stationary.

Another study on the stock market efficiency of the BRICS countries focused on the profitability of trading rules, which infers a deviation from the EMH (Tail Leung Terence, Sam Ho-Sum Cheng, and Elfreda Wong 2010), associated with the simple moving average (SMA), the relative strength index (RSI), the moving average convergence divergence (MACD), and momentum (MOM). While all of these markets are proven to be inefficient, it is found that the indicators are most profitable in the Russian stock market. The Brazilian stock market is found to be the most efficient market among the BRICS countries.

Structural breaks are considered in the study of Kiran and Prabhakar (2019) when examining the weak-form efficiency of BRICS stock markets. They used two methods, one of which is the LM unit test with one and two structural breaks as given by Junsoo Lee and Mark Strazicich (2003). The second method is the ADF-type unit root test with two structural breaks, as proposed by Narayan and Stephan Popp (2010). The LM unit root test with a single break revealed the existence of a unit root for all four indices. In the case of two breaks, the null hypothesis of the unit root for Brazil, India, and China was rejected. The results of the unit root test proposed by Narayan and Popp (2010) for the BRICS stock indices showed that the stock markets of Russia, India, and China had a unit root; for Brazil, the null hypothesis was rejected. The application of the BDS and the K2k test as structural breaks revealed that BRICS stock markets are not weak-form efficient.

Apart from Brazil, Russia, India, China, and South Africa as BRICS countries, the efficiency of the Turkish stock market, Borsa Istanbul, has been frequently analyzed individually and also within the emerging market segment. Most recently, Aliyev (2019) examined the efficiency of Borsa Istanbul by using a smooth transition autoregressive (STAR)-type nonlinear model, nonlinear ARCH and STAR models, and two linear models, which were the linear AR model and random walk model. He examined 10 years of weekly data and then, out-of-sample, forecast the next 12 weeks' return. Comparing the forecast performance powers, he found that the STAR model outperformed random walk; that is, Borsa Istanbul returns are predictable for the given period. He showed that contrary to the linear-level studies, Borsa Istanbul is not weak-form efficient at the nonlinear level within the studied period.

The obtained panel-unit-root test results gave non-stationary results only in linear and TAR-type models. In line with these results, it can be said that there is no TAR-type and linear structure in the dynamics of the markets concerned. In addition, since individuals do not act together in financial markets, it is known that they have nonlinear structures with smooth transitions. In this sense, it is also seen in our study that sudden shifts in the TAR-type do not explain the financial markets. Apart from this, the other tests used in the study are non-linear time-dependent and state-dependent unit root tests with smooth transition dynamics. In this sense, it is apparent that they will be more successful in capturing the dynamics of financial markets. Considering the UO and EO tests, the trend-containing test types achieved stationarity and confirmed the market inefficiency hypothesis. Both of these tests have a state-dependent nonlinear structure. The OSS test is a smooth transition structural break test. Enders and Lee (2012) state that the time series versions of UO and OSS tests are alternatives to each other. The test structure to be described here is the OHS test, which is a smooth

transition structural break test and can be described as a time-dependent non-linear structure. The phenomenon of the sample being stationary with state-dependent and time-dependent non-linear structures was studied in the Yifei Cai and Omay (2021), EOSN (2021), Omay, Emirmahmutoğlu, and Syed Jawad Hussain Shahzad (2021) and Omay and Dumitru Baleanu (2021).

In some cases, time-dependent and state-dependent non-linear structures overlap. The structural break in time also coincides with the state-dependent non-linear structure, thus imitating the data generation process in both test types. In general, agents in financial markets do not act simultaneously, so they cause a smooth non-linear behavior in financial data in time or state.

The results of this study contributed to the strength of nonlinear models over linear ones, specifically in the process of testing the efficiency of stock markets. The stock markets of Turkey, China, Russia, Brazil, and South Africa were selected based on the market capitalization of the stock markets deliberately to form a homogenous panel. Furthermore, from a broader perspective, the results mainly contribute to the empirical proposal that financial variables must consider nonlinearity not to specify their models wrongly, leading to inaccurate conclusions. Moreover, there are still mixed and scanty results on the stock markets under investigation as studies have been using time-series methodologies. However, as explained in the introduction, the panel version of the test suffers from spillover effects, global shocks, and contamination effects. These models mostly used nonlinear time-series unit root tests; however, our contribution in this study is to consider the market spillovers and other common factors by using the panel data structure. In utilizing the panel approach, we have limited misspecifications in modeling correct specifications of the stock markets. In time-series analyses, we have no opportunity to account for global factors such as crisis spillovers or contamination effects of other countries, which will lead to miss-modeled analysis and weak-form efficient results. Using this state-dependent, time-dependent, and hybrid structure in panel unit root testing, we conclude the inefficiency of the markets, which has important policy implications for both policymakers and investors.

## 4. Conclusion

It is generally proposed that in an efficient market, there are many rational participants actively trading to maximize profits. Each of them has a motivation to anticipate the future market price of individual securities. In terms of market dynamics, it has also been proposed that the market prices always and fully reflect all available information. Consequently, no gains can be achieved by the investors upon speculation. Market efficiency can be of three forms: weak, semi-strong, and strong. In this study, the focus is on the weak form of market efficiency where the market prices reflect all past information, using stock market data from five main emerging market countries: Turkey, China, Russia, Brazil, and South Africa.

Although the investment in the stated emerging stock markets promises higher yield than the stocks traded in the developed countries' stock markets, the above-mentioned countries are generally characterized by thin trading causing low levels of liquidity and, in some cases, ill-informed investors with limited access to information,

which could at time be unreliable. This phenomenon could create a view that some stock markets within this group are information-inefficient.

Since the 1990s, several approaches have been developed for testing the weak-form efficiency of stock markets, such as correlation tests, runs tests, and tests based on some trading rules with technical analysis such as the filtering rule, moving average rule, and channel rule/trading range breakout rule. A new wave of research has used linear and nonlinear unit root tests. Other models addressing state-dependent nonlinearity, particularly the model proposed by Ucar and Omay (2009), the model of Emirmahmutoglu and Omay (2014), and the FIPS test of Hasanov and Omay (2018), show that those emerging stock markets as a panel are stationary and inefficient. The results of two other nonlinear methods of Omay, Hasanov, and Shin (OHS 2017) and Omay, Çorakçı, and Emirmahmutoglu (OCE 2017) also indicate the inefficiency of the markets. The results of this study contribute to the strength of nonlinear models over linear ones, specifically in the process of testing the efficiency of stock markets. The stock markets of Turkey, China, Russia, Brazil, and South Africa were particularly selected according to the market capitalization of the stock markets in order to form a homogeneous panel.

Furthermore, from a broader perspective, the results specifically contribute to the empirical proposal that financial variables should also take into account the issue of nonlinearity in order to achieve more accuracy in their overall findings. Moreover, the methodologies employed here have more power concerning their time-series counterparts. This is because they consider the spillover effects, global shocks, and contamination effects, further enhancing the ability to identify market efficiency in light of the size distortions in time-series unit root tests. The new findings lead to recommendations for improved policy-making. The trading rules that address the nonlinearities generating a consistent asymmetrical pattern of return dynamics can be provided over normal returns. Even when there is no nonlinearity in the conditional variance of stock returns, the nonlinear adjustment of stock returns may offer profitable opportunities. However, investing in the stocks of emerging market countries requires careful and selective attention to market dynamics. For the countries discussed in this paper, contamination and spillover may arise from both the global financial markets and the domestic ones.

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