A NEW APPROACH FOR ESTIMATING THE RELIABILITY OF MISSILES

A DOCTOR OF PHILOSOPHY (PhD) THESIS

in

Modeling and Design of Engineering Systems (MODES)

Atılım University

by

YİĞİT KORAY GENÇ

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A NEW APPROACH FOR ESTIMATING THE RELIABILITY OF MISSILES

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Approval of the Graduate School of Natural and Applied Sciences, Atılım University.

____

Prof.Dr. K.İbrahim AKMAN

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I certify that this thesis satisfies all the requirements as a thesis for the degree of Doctor of Philosophy.

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This is to certify that we have read the thesis "A New Approach for Estimating the Reliability of Missiles" submitted by "Yiğit Koray GENÇ" and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Doctor of Philosophy.

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YİĞİT KORAY GENÇ

ABSTRACT

A NEW APPROACH FOR ESTIMATING THE RELIABILITY OF MISSILES

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PhD in Modeling and Design of Engineering Systems (MODES)

Supervisor: Asst.Prof.Dr. Cenk Güray

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Today, reliability is one of the most important bases regarding the performance efficiency of an engineering system. Recently, the complexity of the common engineering systems has been increasing visibly in order to cope with the complexity of the problems these systems are engaged with. The number of components that are used to constitute these systems also has an increasing impact to this situation. This both increases the cost of these systems extensively and also reveals the requirement for a more challenging system due to limitations (weight, dimension etc. Defense industry is one of the areas that the reliability studies are frequently used in. Defense industry contains several examples of complex and expensive engineering systems such as missiles, aircrafts, attack helicopters, battle tanks, etc. As these defense systems are designed for critical missions and are used in harsh operational conditions, the need for durability and performance satisfaction requires the need of a reliability estimation tool design phase. In this thesis, a new fuzzy reliability estimation model which serves as a decision support system for missile systems in design phase is proposed.

Keywords: Reliability, Missiles

ÖZ

FÜZE SİSTEMLERİNİN GÜVENİLİRLİK TAHMİNİ İÇİN YENİ BİR YAKLAŞIM

Genç, Yiğit Koray

Doktora, Mühendislik Sistemlerinin Modellenmesi ve Tasarımı

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Günümüzde güvenilirlik bir mühendislik sisteminin en önemli performans göstergelerinden biri haline gelmiştir. Son yıllarda, problemlerin karmaşıklığı arttığından, bu problemleri ele alacak mühendislik sistemlerinin de karmaşıklığı artmaktadır. Buna göre sistemleri oluşturan bileşen sayılarında artış görülmektedir. Belirtilen her iki etken de maliyet artışına sebep olmakta ve ağırlık, boyut vb. kısıtlamalar çerçevesinde daha rekabetçi ürünler ortaya koymayı gerektirmektedir. Savunma sanayi güvenilirlik çalışmalarının sıkça kullanıldığı bir alandır. Bu alanda füze, uçak, helikopter, tank sistemleri gibi son derece karmaşık ve pahalı sistemler yer almaktadır. Zorlu operasyon koşullarında kendilerine atanan kritik görevleri yerine getirmek için tasarlanan bu sistemlerin dayanıklılık ve performans gereksinimleri, bu sistemlerin daha güvenilir tasarlanabilmeleri için güvenilirlik tahmini yapılmasına olanak sağlayacak araçlar gerektirmektedir. Bu tezde, füze sistemlerinin henüz tasarım aşamasında iken güvenilirliğini tahmin etmeyi sağlayacak ve karar destek sistemi olarak kullanılabilecek yeni bir bulanık tahmin yöntemi önerilmektedir.

Anahtar Kelimeler: Güvenilirlik, Füzeler

To My Parents

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LIST OF ABBREVIATIONS

MTBF	-	Mean Time Between Failures
MOM	-	Middle of Maximum
LOM	-	Largest of Maximum
SOM	-	Smallest of Maximum
BOM	-	Bill of Material
λ	-	Failure Rate
MTBM	-	Mean Time Between Maintenance
MTBR	-	Mean Time Between Repair
MTBCF	-	Mean Time Between Critical Failure
MTBOMF	-	Mean Time Between Operational Mission Failure
MTTF	-	Mean Time To Failure
$F(t)$	-	Cumulative probability function
$R(t)$	-	Reliability function
$f(t)$	-	Probability density function
$h(t)$	-	Hazard rate
RBD	-	Reliability Block Diagram
MRS	-	Mobile Robot System
FMEA	-	Failure Modes Effects Analysis
FMECA	-	Failure Modes, Effects and Criticality Analysis
FTA	-	Fault Tree Analysis
WCCA	-	Worst Case Circuit Analysis
FEA	-	Finite Element Analysis
SCA	-	Sneak Circuit Analysis
GMP	-	Generalized Modus Ponens
GMT	-	Generalized Modus Tollens

CHAPTER 1

INTRODUCTION

Today's engineering systems are more complicated than ever before. Since these systems are getting more complicated, consumers' purchasing behavior changed to not only considering the cost and functionality, but also durability of the system that they are going to buy as well. In today's competitive market, companies try to find cost-effective solutions to improve product reliability while increasing their profitability. By focusing on defect prevention rather than defect removal is one of the best approaches to achieve this goal. According to Smith [1], a single defect can easily cost £100 in diagnosis phase to repair if it is detected early in production whereas the same defect in the field may well cost £1000 to rectify. If the failure is caused by the design fault then the cost of redesign, documentation and retest may well be in tens or even hundreds of thousands of pounds [1].

The utility of a product as perceived by the customer is an important issue for the preferability of the brand. This utility is measured in terms of ease and economy of construction, reliability, serviceability, availability, functionality, aesthetic appeal and price. Reliability is also important from the quality point of view. That is why reliability, quality and safety of systems become the most important design criteria as an addition to the cost of the design for today's engineering systems.

1.1. Importance of Missile Reliability

Defense industry contains highly complex engineering systems such as missiles, aircrafts, attack/utility helicopters, battle tanks, etc. They are very expensive and their applications in the military field are crucially important. Defense systems are used in the harsh operational conditions. For defense systems, Mission Reliability can be defined as the ability of an item to perform its required functions and deliver the required performance for the duration of a specified mission profile. The requirement for high mission reliability in these harsh operational conditions reveals the need for a defense system more powerful than the others. For this reason, reliability is one of the most important design criteria for defense systems.

Especially, complex missile systems require considerable resources such as time, money and manpower to achieve a certain level of system reliability because they are one-shot systems and it is almost not possible to use pre-used missiles. Also number of tests on missiles is strongly limited since some reliability tests may preclude the further use of the test article. Thus, there is a desire to obtain reliability estimations with a limited amount of data in design phase in order to minimize number of tests (consequently cost) needed to establish a reliability estimate with a desired degree of confidence.

1.2. Problem Definition

Mission reliability of missile system at different phases of its life cycle such as storage, transportation, firing, etc. is critical and for this reason the reliability estimations at these phases should be handled in an efficient way during system design phase. Since the missile systems are expensive and one-shot systems, testing a determined number of missiles to get the reliability estimation is not practical and generally not applicable within short project periods and limited budgets. A more practical way is required to estimate reliabilities of missiles during design phase to improve the design reliability if required.

1.3. Aim and Scope

Classical reliability prediction techniques require reliability modeling via reliability block diagrams, finding the failure rates of all the components in Bill-of-Materials by using reliability libraries and finding the reliability of the system at a time. However, developing the Bill-of-Materials in enough detail at further steps of design phase to select all the components from the reliability libraries require much work, also big effort is paid to accomplish this task in short project periods. Also, the reliability libraries include limited number of components and, very often, the special components used in missile systems can't be matched with the components available in the reliability libraries. This causes to select a similar component from the reliability library instead of the one used in the design causing a somewhat decrease in the reliability analysis fidelity.

To overcome these disadvantages and propose an alternative way for estimating the system reliability at design phase, a fuzzy method which is based on Failure Modes Effects Analysis (FMEA) is studied in this thesis. According to this, expert opinions are considered via FMEA and by using the expert opinions, a fuzzy logic system is constructed to get the reliability assessment of the missile system under development and to relatively observe the effects of possible design changes on reliability.

CHAPTER 2

LITERATURE SURVEY

2.1. General Survey on Reliability

1920s and 1930s were the years that new complicated systems are met such as telephones and electron vacuum tubes. Increasing demands for these new products required a more economical and reliable manufacturing. These two new technologies spurred early reliability studies [2].

During World War II, airborne radios that were delivered into remote theaters of war had demanded reliability. It is seen that only ~17% of the remote theaters worked upon arrival into the battle zone. It is a proof of not considering the reliability concept for such systems during those periods [2].

Also V-1 rockets were developed at these years. The V-1 rocket had a demonstrated reliability of 1 success out of 11 attempts, that is a calculated reliability of 9.1%. This was a great result for frontier technology but a terrible success rate considering the consumption of limited resources [2].

Robert Lussor was the first researcher to quantify reliability studies of V-1 rockets. He used principles learned from the study of electron vacuum tube reliability. His studies resulted in Werner Von Braun’s redesign leading to the V-2 rocket. The V-2 rocket used the principles of “redundancy” to enhance the rocket’s reliability. The V-2 rocket results are written in the history books for a demonstrated reliability improvement program that resulted in the building of more than 8000 V-2 rocket motors within this reliability concept [2].

The Korean War (1950-1953) was a war of new technological products such as gas turbines, helicopters, miniature electron vacuum tubes, etc [2]. With the growing technology, complex systems were delivered and the attention on reliability has also increased.

Advisory Group on Reliability of Electronic Equipment (AGREE) was established by American Defense Ministry during these years. AGREE studies introduced that $2 of maintenance costs were spent for every $1 of capital for electronic products during the Korean War. After realizing this fact, products with higher design-manufacturing cost but with less maintenance cost became preferable.

High maintenance costs led to establishment of reliability requirements for procurement of military equipment and new military standards were developed at the beginning of 1960’s [2]. The first text books were written for the emerging field of reliability during the early 1960’s as a spin-off of NASA activities for manned space programs. During this period some claimed NASA could identify every rocket failure but could not correct reliability problems. This embarrassing reliability situation encouraged the use of reliability engineering principles which quickly achieved higher successes [2].

During the 1960s, ‘70s, and ‘80s applications of reliability principles increased with respect to the previous years. Performance improvement and cost reduction programs occurred in mainframe computers, gas turbine engines, nuclear reactors, electronics, automobiles, and consumer products using reliability engineering principles [2].

During the 1990s, industries such as petrochemical and refining began to be active by means of formal programs to improve reliability and decrease costs [2]. Highly competitive market environment triggered this improvement. The requirement for professional staffs to apply reliability engineering techniques appeared.

Today, acceleration of the attention on reliability is still increasing due to complexity of the systems and competitive market environments.

There are several studies in the literature on system reliability. They can be mainly categorized according to the problem type and optimization technique. Kuo et al. [3] categorized the system reliability studies with respect to the problem type and the optimization technique. Summary for classification of the system reliability studies by problem type and the optimization technique can be seen in **Figure 1** and **Figure 2** respectively [3].

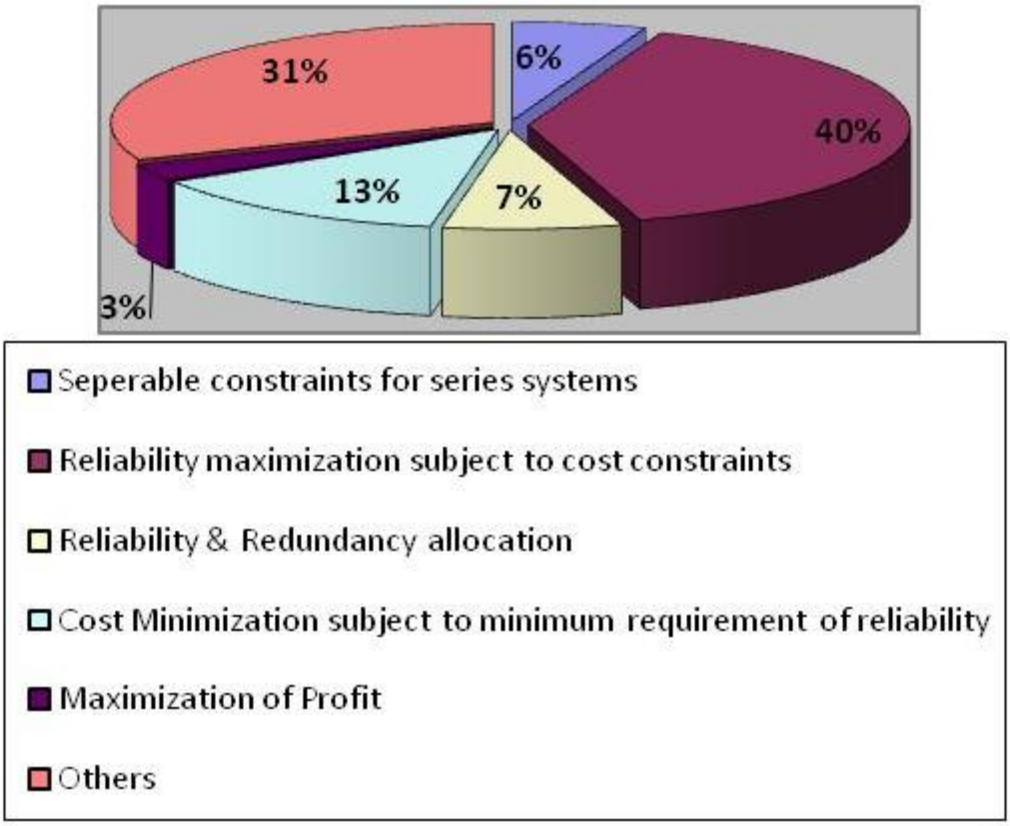


Figure 1: Classification of the system reliability studies by problem type

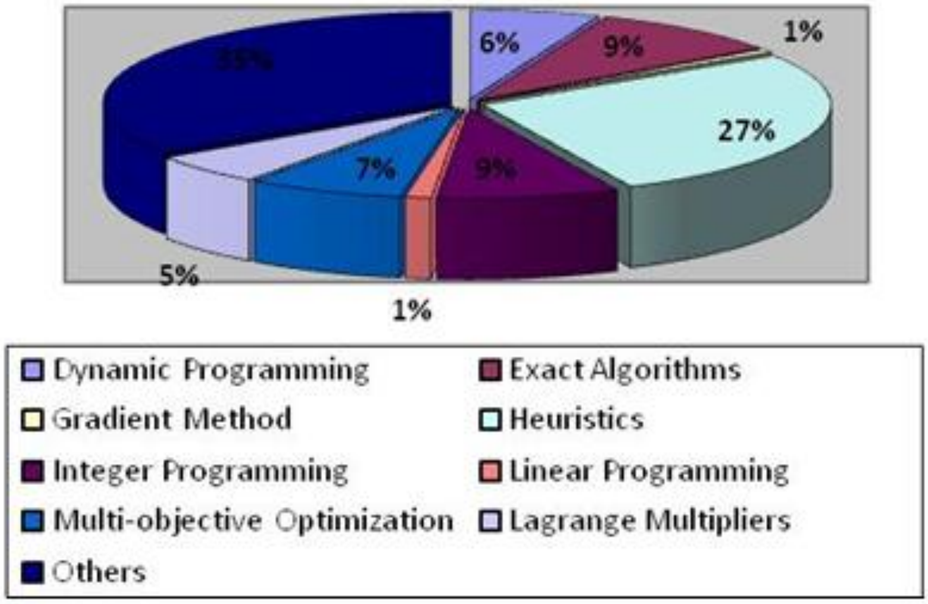


Figure 2: Classification of the system reliability studies by optimization technique

According to **Figure 1** and **Figure 2**, it can be stated that maximization of system reliability subject to cost constraints is the most commonly solved problem type and heuristic applications are commonly used methods for optimization in system reliability literature.

2.2. Survey on Missile Reliability

2.2.1. Literature for Missile Reliability

Studies on missile reliability are very rare in open literature. First studies on this issue can be found in the early 90s. Limited number of studies exists on missile reliability because of limited number of experts in this area, the addiction to the traditional design understanding, high cost impacts and short project schedules. Also, such information is often regarded as confidential. It is also observed that the number of national studies on missile reliability has not been many.

The main purpose of the missile reliability studies is to use the existing limited data as much as possible to estimate the reliability of the missile. Different techniques are applied to get the best estimation on missile reliability. In **Table 1**, some of the well-known studies on missile reliability are listed and the purposes of the studies, the methods used and the application details are given.

Table 1: Literature Survey on Missile System Reliability

Year	*Title*	*Purpose*	*Method*	*Application*
1990	Automated reliability life data analysis of missiles in storage and flight [4]	To describe the methodology underlying a computer tool known as RAMFIT, which was developed to perform automated life data analysis on missiles using either interval data for missiles in storage, or continuous data for missiles in the captive carry environment.	Maximum Likelihood equations, Goodness of Fit	Example data interval is used for illustration.
1996	A Reliability Prediction Method for Missile Systems Based on Truncated Weibull Distribution [5]	A reliability prediction model was developed for the systems under development.	Reliability prediction models based on a truncated Weibull distribution both time and failure truncated cases.	Applied on sample missile systems.
1997	A Statistically Based Approach for Predicting Stockpiled Missile System Reliability [6]	To develop a statistically based approach for predicting the reliability of stockpiled missile systems stored under normal conditions through storage life.	Using regression analysis and appropriate statistical hypothesis tests, a model is developed for predicting missile stockpile reliability	Applied on a sample for missile systems aged 5 through 14.

Year	*Title*	*Purpose*	*Method*	*Application*
1997	Missile Reliability Analysis with Censored Data [7]	The use of an exponential distribution as a model for the captive-carry life length of a missile system is explored.	Comparison between Kaplan-Meier estimator and Exponential Distribution.	Applied on a sample censored data.
2002	Ultrahigh Reliability US Army Missiles and Munitions [8]	To provide an overview of US Army requirements necessary for missiles and munitions to survive and operate in extreme environments with ultrahigh reliability.	Overview	
2003	Using Neural Networks For Estimating Cruise Missile Reliability [9]	To develop a predictive model that delivers a realistic 24-month reliability projection.	Neural Networks	Applied neural networks for estimating 24-month reliability of a cruise missile by using existing data sources.
2005	A comparison of reliability estimation methods for binary systems [10]	To estimate the probability that a system (e.g. a space launch vehicle or intercontinental missile) will work when called upon based on a limited number of tests on similar systems.	Two classical (frequentist) methods, two Bayesian methods, and the method used by the US Air Force Strategic Command (STRATCOM)	Five different approaches tested and compared for estimating the reliability of a complex binary system based on limited test results.
2005	A hierarchical model for the reliability of an Anti-aircraft missile system [11]	To estimate anti-aircraft missile reliability based on multiple sources of information, including component, subsystem and	Bayesian hierarchical model	Applied on a small size anti-aircraft missile

Year	Title	Purpose	Method	Application
		system data, as well as prior expert opinion		

Warren and Robins [4] studied on the reliability life data analysis of missiles in storage and flight phases. The aim of their study is to find the cause of the failures according to the periodic tests performed at storage and flight phases. For finding a solution to this problem, they developed a computer tool called as RAMFIT.

Hwang [5] studied on a reliability prediction model for missile systems based on a truncated Weibull distribution. Time and failure truncated Weibull model is developed for reliability analysis of missiles at development test phase. Also the model proposed is applicable to other systems as well.

West [6] developed a statistical based approach for assessing the reliability of stockpiled missile systems stored under normal conditions utilizing existing historical live fire test data from surveillance programs.

Williams and Pohl [7] focused on the problem of estimating the captive-carry life length of a missile, which is a measure of the cumulative time a missile can be carried by an aircraft in-flight and still the launch can remain capable, from highly censored data.

Erickson, Shankle and Marotta [8] overviewed the U.S. Army requirements for high reliability and minimum field checks, which is called as ultra-reliability. Ultra-reliability concept is further discussed in the study.

Hoffman [9] studied on a predictive model for free flight reliability of cruise missile systems. Neural Network theory is used in the model for the 24-month projection of cruise missile reliability which relies upon past ground and flight test pass rates to estimate reliability.

Guikema [10] developed a simulation-based approach to compare the predictive accuracy of five different methods for estimating the risk of failure for binary failure/no failure systems such as U.S. strategic missiles, space launch vehicles, and security systems based on the results of a number of tests.

Reese et. al. [11] proposed a framework for assessing the reliability of multi-component systems by addressing two important analytical concerns: the integration of available information at various levels to assess system reliability, and estimating reliability growth or degradation. Failure time data are collected from experts and the system at either the component or subsystem level, aggregated into the posterior distribution, and the pooling of failure information between similar components. Weibull distribution is used to model the component failure time on a sample problem.

As Reese et. al. [11] underlined, many common reliability models are not able to account for prior expert opinions. Most of the missile system reliability studies in the literature are based on the past /test data results. Reese et.al.'s study [11] is taking the expert opinions into account for estimating the reliability of missiles. However, instead of using failure time data of system/subsystems as expert opinions, it is possible to develop a model which provides inferencing based on the detailed expert opinions concerning all the failure modes and their effects at all the phases of the missile systems. For this purpose, a systematic and efficient way is required to handle expert opinions. In this thesis, detailed expert opinions are taken into account in a systematic way and inferencing is provided.

CHAPTER 3

RELIABILITY MATHEMATICS

3.1. Definition of Reliability

There are several definitions of reliability in the literature. Mainly reliability is the probability of equipment or processes to perform its intended functions without failure when operated correctly for a given interval of time under stated conditions [13].

Definition of reliability depends on the repair-ability of the system. For repairable items, reliability can be defined as the probability of an item for performing its intended function for a specified interval under stated conditions [14]. For non-repairable items, reliability can be defined as the probability that an item will perform a defined function without failure under stated conditions for a stated period of time [14]. In this instance the purpose of the studies is to increase the operational duration as long as possible.

Sometimes system effectiveness, availability, durability, operational readiness and reliability terms are confused. Availability is a measure of degree to which an item is an operable and committable state at the start of a mission when the mission is called for at an unknown (random) time [16]. If system is not repairable, availability is same as reliability. Durability is a measure of an item's useful life [16]. System effectiveness is the compound probability of performance, availability and reliability. Operational readiness is the ability of a unit to respond its operation plans upon request of an operations order [16].

3.1.1. Reliability Characteristics

Reliability of a component or system at time t is denoted as $R(t)=P(T>t)$, where T represents the lifetime (time to failure) of the component or the system. The unreliability $F(t)= 1-R(t)$ of a component or system is defined as the probability that the component or system experiences the first failure before time t.

There are several characteristics used in system reliability. **Table 2** includes the characteristics and their definitions [12].

Table 2: Reliability Characteristics and Definitions

Parameter	Description
Failure Rate	The total number of failures within an item population, divided by the total time expended by that population, during a particular measurement interval under stated conditions.
Mean Time Between Failure (*MTBF*)	A basic measure of reliability for repairable items. The average time during which all parts of the item perform within their specified limits during a particular measurement period under stated conditions.
Mean Time Between Maintenance (*MTBM*)	A basic measure of reliability for repairable fielded systems. The average time between all system maintenance actions. Maintenance actions may be for repair or preventive purposes.
Mean Time Between Repair (*MTBR*)	A basic measure of reliability for repairable fielded systems. The average time between all system maintenance actions requiring removal and replacement or repairs of a

	box or subsystem.
Mean Time Between Critical Failure (*MTBCF*)	A measure of system reliability that includes the effects of any fault tolerance that may exist. The average time between failures that cause a loss of a system function defined as "critical" by the customer.
Mean Time Between Operational Mission Failure (*MTBOMF*)	A measure of operational mission reliability for the system. The average time between operational mission failures which cause a loss of the system's "mission" as defined by the customer. This parameter may include both hardware and software failures.
Mean Time To Failure (*MTTF*)	A basic measure of reliability for non- repairable systems. Average failure free operating time, during a particular measurement period under stated conditions.

Failure Rate for items may deviate over time. Bathtub Curve can be used for observing the deviation of the Failure Rate over time as given in **Figure 3** [38].

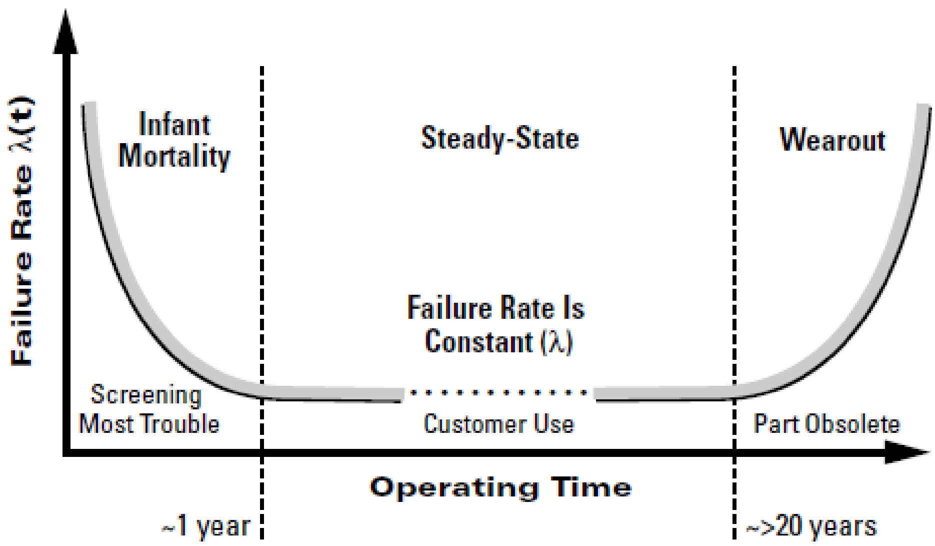


Figure 3: Bathtub Distribution

The relation between *MTBF* and the failure rate is given in Eq.(1) and the equation holds true only if $\lambda(t)=\lambda$, i.e. when failure rate is constant.

$$MTBF = \frac{1}{\lambda} \qquad (1)$$

Reliability-Investment Cost Relation can be seen in **Figure 4**. If higher reliability is desired, investment costs increase accordingly.

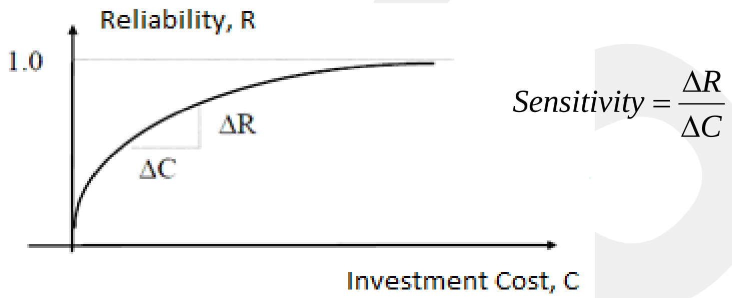


Figure 4: Reliability-Investment Cost Relation

Reliability-Total Cost Relation can be seen in **Figure 5**. As seen from the figure, reliability and total cost has an optimum point. High reliability values increases the investment cost and total cost as well, however the total cost function has its lowest value at a certain level of reliability.

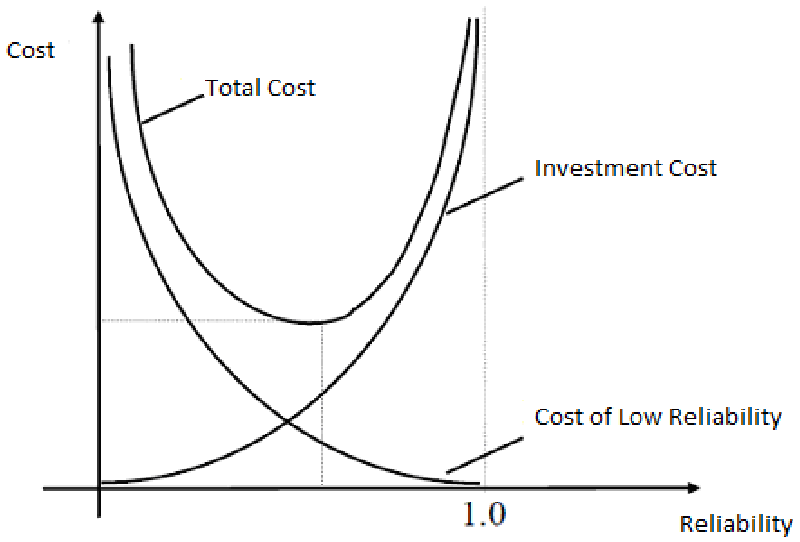


Figure 5: Reliability-Total Cost Relation

3.2. Reliability Mathematics

Since reliability is based on probability of failures, statistical relations are applicable on reliability theory. In general, the reliability of a system consisting of *n* components is a function of its components reliabilities and can be represented as;

$$R(t) = h\big(R_1(t), \ldots, R_n(t)\big) \quad for\ t \geq 0, \qquad (2)$$

where $R_i(t)$ is the reliability of the i^{th} component, $i=1,\ldots,n$.

If for example, the system has a series structure and consists of independent components, then

$$R(t) = R_1(t) \ldots R_n(t) \quad for\ t \geq 0. \qquad (3)$$

3.2.1. Functions Used in Reliability Analysis

3.2.1.1. Cumulative Distribution Function (*F(t)*)

If *T* denotes the time to failure of a system, then the cumulative distribution function associated with *T* is defined as follows [18];

$$F(t) = P(T \leq t) \quad for\ t \geq 0 \qquad (4)$$

If *F* is continuous, then we have;

$$F(t) = \int_0^t f(x)dx \quad for\ t \geq 0 \qquad (5)$$

where *f(t)* is the probability density function satisfying

$$\int_0^\infty f(t)dt = 1 \quad for\ t \geq 0. \qquad (6)$$

3.2.1.2. Probability Density Function

The probability density function *f(t)* describes the probability for a component failure in different time intervals during the component service time. This function can be utilized to determine the probability that a failure takes place in a given time interval. Eq. (7) and Eq. (8) shows the relations of *f(t)* with *F(t)* and *R(t)* respectively [15], [18].

$$f(t) = \frac{dF(t)}{dt} \quad for\ t \geq 0, \qquad (7)$$

$$f(t) = -\frac{dR(t)}{dt} \quad for\ t \geq 0. \qquad (8)$$

3.2.1.3. Reliability Function

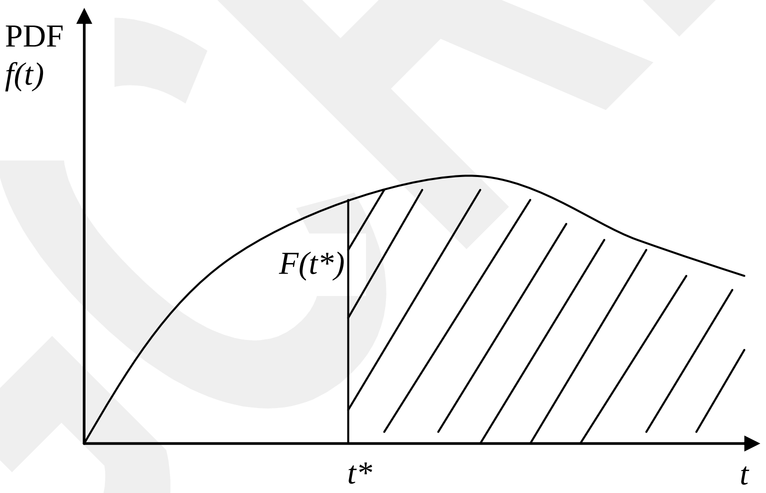


Figure 6: Reliability Function

F(t)* shown in **Figure 6** is the probability that component has failed before time *t*. Reliability at time *t* can be calculated as given in Eq.(9) [18].

$$R(t) = \int_{t}^{\infty} f(x)dx \quad for\ t \geq 0. \qquad (9)$$

For a system with a constant failure rate, reliability can be calculated as given in Eq.(10) [15].

$$R(t) = e^{-\lambda t} \quad for\ t \geq 0. \qquad (10)$$

3.2.1.4. Hazard (Failure) Rate (*h(t)*)

Hazard rate or failure rate can be defined as the ratio of the number of failures in a given time period to the number of components exposed to failure [18]. Hazard rate is denoted as *h(t).* The hazard rate (or time dependent failure rate) of a component, a sub-system, or a system is given by Eq.(11);

$$h(t) = \frac{f(t)}{R(t)} \quad for\ t \geq 0. \qquad (11)$$

Relation given in Eq.(12) holds true between the lifetime distribution and the corresponding failure rate function [18], [16].

$$F(t) = 1 - exp\left[-\int_0^t h(t)dt\right] \quad for\ t \geq 0. \qquad (12)$$

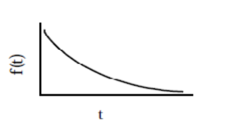
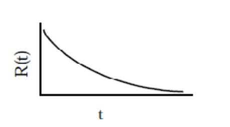
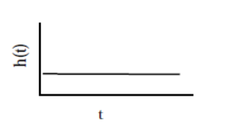
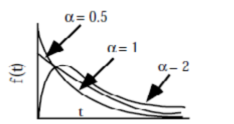
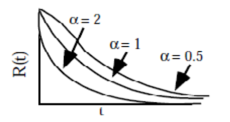
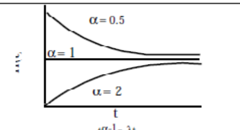
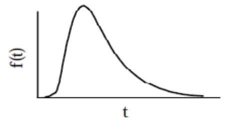
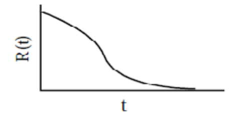
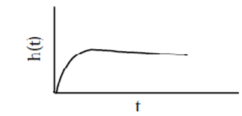
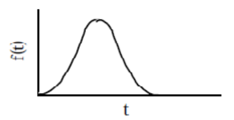
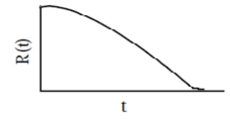
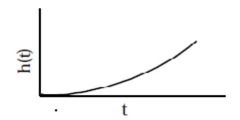
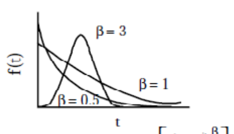
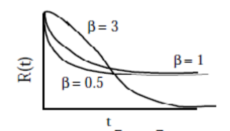
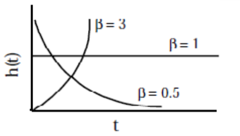
If the system has a constant failure rate, then they can be described by an exponential distribution and hence the reliability can be computed from Eq.(13).

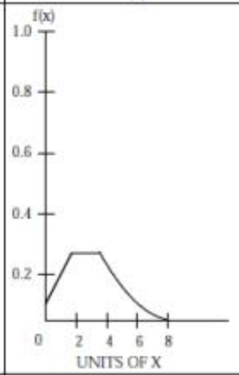
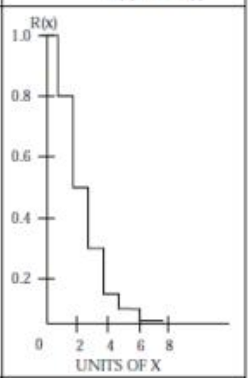
$$R(t) = e^{-\lambda t} = e^{\frac{-t}{MTBF}} \quad for\ t \geq 0. \qquad (13)$$

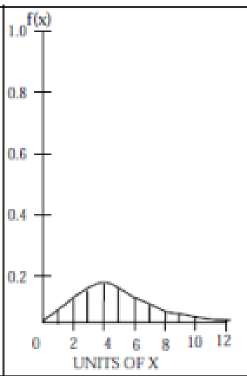
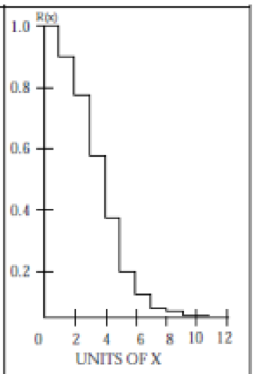
3.2.2. Some Commonly Used Probability Distributions in Reliability

Probability distributions frequently used in reliability prediction with their *f(t), R(t)* and *h(t)* are given in **Table 3** [16].

Table 3: Summary of the frequently used distributions in reliability

TYPE OF DISTRIBUTION	PROBABILITY DENSITY FUNCTION, f(t)	RELIABILITY FUNCTION $R(t) = 1 - F(t)$	HAZARD FUNCTION $h(t) = \frac{f(t)}{R(t)}$
EXPONENTIAL	 $f(t) = \lambda e^{-\lambda t}$	 $R(t) = e^{-\lambda t}$	 $h(t) = \lambda = \theta^{-1}$
GAMMA	 $f(t) = \frac{\lambda}{\Gamma(\alpha)} (\lambda t)^{\alpha-1} e^{-\lambda t}$	 $R(t) = \frac{\lambda^{\alpha}}{\Gamma(\alpha)} \int_t^{\infty} t^{\alpha-1} e^{-\lambda t} dt$	 $h(t) = \frac{t^{\alpha-1} e^{-\lambda t}}{\int_t^{\infty} t^{\alpha-1} e^{-\lambda t} dt}$
LOGNORMAL	 $f(t) = \frac{1}{\sigma t \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln t - \mu}{\sigma}\right)^2}$	 $R(t) = 1 - \phi\left(\frac{\ln t - \mu}{\sigma}\right)$ See Note	 $h(t) = \frac{f(t)}{1 - \phi\left(\frac{\ln t - \mu}{\sigma}\right)}$
NORMAL	 $f(t) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t - \mu}{\sigma}\right)^2}$	 $R(t) = 1 - \phi\left(\frac{t - \mu}{\sigma}\right)$ See Note	 $h(t) = \frac{f(t)}{1 - \phi\left(\frac{t - \mu}{\sigma}\right)}$
WEIBULL	 $f(t) = \frac{\beta}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{\beta-1} e^{-\left[\left(\frac{t-\gamma}{\eta}\right)^{\beta}\right]}$	 $R(t) = e^{-\left[\frac{(t-\gamma)^{\beta}}{\eta}\right]}$	 $h(t) = \frac{\beta}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{\beta-1}$

TYPE OF DISTRIBUTION	PARAMETERS	PROBABILITY DENSITY FUNCTION f(x)	RELIABILITY FUNCTION $R(x) = 1 - F(t)$
BINOMIAL	MEAN, $\mu = np$ Standard Deviation, $\sigma = \sqrt{npq}$ $\binom{n}{x} = \frac{n!}{(n-x)!x!}$ $q = 1 - p$		
	Sample data used to plot charts shown	$f(x) = \binom{n}{x} p^x q^{n-x}$ $\left\{\begin{matrix} n=8 \\ p=2/3 \end{matrix}\right\}$	$R(x) = \sum_{i=x}^{n} \binom{n}{i} p^i q^{n-i}$ $\left\{\begin{matrix} n=8 \\ p=2/3 \end{matrix}\right\}$

POISSON	MEAN, $\mu = a$, Standard Deviation, $\sigma = \sqrt{a} = \sqrt{\lambda t}$		
	Sample data used to plot charts shown	$f(x) = \frac{a^x e^{-a}}{x!} = \frac{(\lambda t)^x e^{-\lambda t}}{x!}$, $a = \lambda t = 4$	$R(x) = \sum_{i=x}^{\infty} \frac{a^i e^{-a}}{i!} = \sum_{i=x}^{\infty} \frac{(\lambda t)^x e^{-\lambda t}}{x!}$, $a = \lambda t = 4$

3.2.2.1. Exponential Distribution

Exponential distribution is the most widely used probability distribution in reliability engineering. This is probably the most important distribution in reliability work due to its simplicity and is used almost exclusively for reliability prediction of electronic equipment [16]. Relations between *f(t)*, *h(t)* and *R(t)* are explained below [14], [15].

$$f(t) = \lambda e^{-\lambda t} \quad for\ t \geq 0, \qquad (14)$$

$$F(t) = 1 - e^{-\lambda t} \quad for\ t \geq 0, \qquad (15)$$

$$h(t) = \lambda \quad for\ t \geq 0, \qquad (16)$$

$$R(t) = e^{-\lambda t} \quad for\ t \geq 0, \qquad (17)$$

$$MTBF = \frac{1}{\lambda}\ . \qquad (18)$$

Reliability trajectory over time for exponential distribution is shown in **Figure 7** [33].

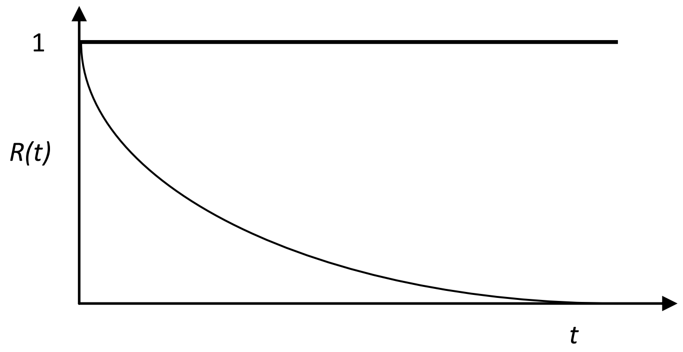


Figure 7: Reliability Trajectory in Exponential Distribution

3.2.2.2. Gamma Distribution

The gamma distribution is used in reliability analysis for cases where partial failures can exist, i.e., when a given number of partial failures must occur before an item fails (e.g., redundant systems) or the time to second failure when the time to failure is exponentially distributed [16]. *R(t), h(t)* and *f(t)* for gamma distribution are given in **Table 3**.

3.2.2.3. Bathtub Distribution

It is generally accepted that electronic systems or components exhibit constant failure rates during the useful operating life. Bathtub distribution is graphically shown in **Figure 3**.

3.2.2.4. Weibull Distribution

The Weibull distribution is particularly useful in reliability work since it is a general distribution which, by adjustment of the distribution parameters, can be made to model a wide range of real life distribution characteristics of different classes of engineering items [16]. Weibull distribution is generally applied for mechanical component or system reliability. *R(t), h(t)* and *f(t)* for Weibull distribution are given in **Table 3**.

3.2.2.5. Binomial Distribution

The binomial distribution is used for those situations in which there are only two outcomes, such as success or failure, and the probability remains the same for all trials. Binomial distribution is a discrete probability distribution like Poisson distribution. They both have a finite number of possible values. It is very useful in reliability and quality assurance work [16]. For binomial distribution, $f(x)$ and $R(x)$ are given in **Table 3**.

3.2.2.6. Poisson Distribution

The probability that a certain number of random events occur in a given unit of time or space can be calculated using a Poisson probability distribution. This distribution is used quite frequently in reliability analysis of sequentially redundant systems [15]. It can be considered an extension of the binomial distribution when n is infinite [16]. For poisson distribution, $f(x)$ and $R(x)$ are given in **Table 3**.

As a result, it can be seen that for reliability characteristics of system components, different probability distributions are available and selection of the best fitting probability distribution for representing the reliability of a component is also another problem to be handled.

3.3. Reliability Analysis Phases

Reliability analysis is composed of three main phases; modeling, allocation and prediction. These phases are explained below.

3.3.1. Reliability Modeling via Reliability Block Diagrams

A Reliability Block Diagram (RBD) performs the system reliability and availability analyses on large and complex systems using block diagrams to show network relationships. The structure of the reliability block diagram defines the logical interaction of failures within a system that are required to sustain the system operations.

RBD starts with an input node located at the left side of the diagram. The input node flows to arrangements of series or parallel blocks that conclude to the output node at the right side of the diagram. A diagram should only contain one input and one output node.

The RBD system is connected by a parallel or series or mixture of both configurations. These configurations are explained below;

3.3.1.1. Series Structure:

Series structure represents a system with *m* independent elements connected in series. In series systems, if any one of the elements fails, the entire system fails. Series structure is represented in **Figure 8** and following equation formulizes the reliability of the system depending on the series units [14].

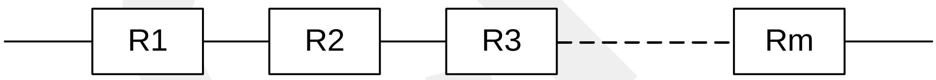


Figure 8: Series Structure

Reliability of systems in series structure can be calculated as given in Eq.(19), where R_i is the reliability of component *i*, and R_s is the series structure reliability.

$$R_s = \prod_{i=1}^{m} R_i \ . \qquad (19)$$

3.3.1.2. Parallel Structure:

Parallel structure is used to represent a system with *m* active and independent elements connected in parallel. These types of systems are also called as redundant systems. In parallel systems, the entire system fails if all the parallel elements fail. Parallel structure is represented in **Figure 9** [14].

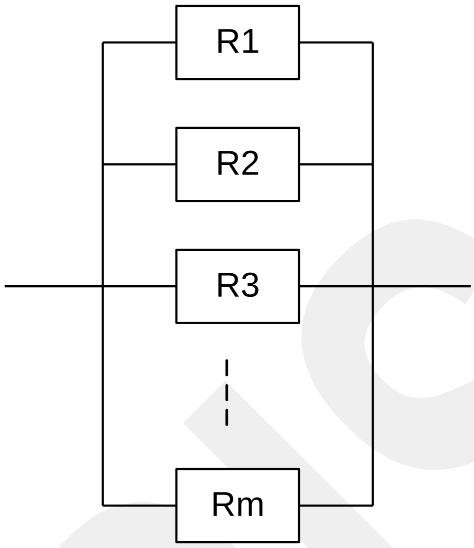


Figure 9: Parallel Structure

Reliability of systems in parallel structure can be calculated as given in Eq.(20), where R_i is the reliability of component i, and R_p is the parallel structure reliability.

$$R_p = 1 - \prod_{i=1}^{m} (1 - R_i) \,. \qquad (20)$$

3.3.1.3. *n*-out-of-*m* Structure:

An *n-out-of-m* system is a system that consists of *n* elements and functions if at least *n* components function. Its reliability can be computed from Eq.(21),

$$R_{n/m} = \sum_{i=n}^{m} \binom{m}{i} R^i \, (1 - R)^{m-i}. \qquad (21)$$

where R is the common reliability of components.

3.3.2. Reliability Allocation

In complex systems, it is necessary to translate system reliability requirements into detailed specifications for the numerous components that make up the system. This process is called reliability allocation or reliability apportionment. The primary purpose of reliability apportionment is to establish a reliability goal or objective for each component so that the personnel responsible for the component's development are made aware of its required reliability. The widely known apportionment methods are listed below [18];

- Equal apportionment method
- ARINC apportionment method
- AGREE apportionment method
- Boyd apportionment method
- Feasibility of objective method
- Dynamic programming approach
- Minimum effort algorithm
- etc.

Most frequently used apportionment methods are explained below;

3.3.2.1. Equal Apportionment

This approach is used for a series system of *n* independent components where we do not have any specific information about the various components. The system reliability requirement is given in Eq.(22).

$$\prod_{i=1}^{n} R_i^* \geq R^*. \qquad (22)$$

By the Equal apportionment method, each component is assigned to the same reliability requirement as given in Eq.(23).

$$R_i^* = (R^*)^{\frac{1}{n}}. \qquad (23)$$

3.3.2.2. ARINC apportionment method

The ARINC method is restricted to series systems of *n* independent and non-repairable components with constant failure rates.

New $\lambda_i = w_i\lambda^*$ where;

$$w_i = \frac{\lambda_i}{\sum_{i=1}^{n} \lambda_i}, \qquad i = 1,2,..,n \qquad (24)$$

and λ^* is the target failure rate.

This means that allocated failure rates are proportional with current failure rates.

3.3.2.3. AGREE apportionment method

This approach allocate equal share of the reliability to each module in the system. The i^{th} component contribution to system reliability is given by $[R^*(t)]n_i/N$. This leads to [18];

$$w_i\left(1 - e^{-\lambda_i t_i}\right) = 1 - [R^*(t)]^{\frac{ni}{N}} \quad \textit{for } t>0. \qquad (25)$$

The left side is the joint probability that i^{th} component fails and results in a system failure. The right side of the failure probability allocated to the i^{th} component. Solving for λ_i result in:

$$\lambda_i = -\frac{1}{t_i} ln\left(1 - \frac{1 - R^*(t)^{\frac{ni}{N}}}{w_i}\right) \quad \textit{for } t>0, \qquad (26)$$

such that;

$$\prod_{i=1}^{n} e^{-\lambda_i t_i} \leq R^*(t), \qquad (27)$$

where;

t is system operating time

$R^*(t)$ is system reliability goal

n is number of components

n_i complexity number, number of modules within component i

N = sum of n_i, total number of modules in system

t_i is operating time of i^{th} component

λ_i is failure rate of i^{th} component

w_i is probability that the system will fail given component i failed.

3.3.3. Reliability Prediction

Reliability prediction is the process of forecasting, from available failure-rate information, the realistically achievable reliability of a part, component, subsystem, or system and probability of its meeting performance and reliability requirements for a specified application [15].

Reliability prediction is a process which is synchronized with system design. Therefore, prediction techniques have different levels depending upon the depth of knowledge of the design and the availability of historical data on equipment and component part reliabilities. There are four reliability prediction techniques which are explained below;

3.3.3.1. Similar Item Analysis Prediction Method

The similar item prediction method utilizes specific experience on similar items. The more rapid way of estimating reliability is to compare the item under consideration with a similar item whose reliability has previously been determined by some means and has undergone field evaluation. The method has a continuing and meaningful application for items undergoing orderly evolution.

3.3.3.2. Parts Count Analysis Prediction Method

This method uses the failure rates of the individual elements in a higher-level assembly to calculate the assembly failure rate.

Parts Count Analysis Prediction Method requires less information, generally part quantities, quality level and the application environment. The Parts Count Methods usually result in a more conservative estimate of system reliability than the Parts Stress Analysis Prediction Method.

This method is used in the preliminary design stage when the number of parts in each generic type class such as capacitors, resistors, etc., is reasonably fixed and the overall design complexity is not expected to change appreciably during later stages of development and production. This method assumes the time to failure of the parts is exponentially distributed (i.e., a constant failure rate). The item failure rate can be determined directly by the summation of part failure rates if all elements of the item reliability model are in series or can be assumed in series for purposes of an approximation [16].

3.3.3.3. Part Stress Analysis Prediction Method

This technique is based upon a knowledge of the stress to which the part will be subjected, e.g., temperature, humidity, vibration, etc., and the effect of those stresses on the part's failure rate.

Part Stress Analysis Method requires a greater amount of detailed information and is applicable during the later design phase when actual hardware and circuits are designed.

3.3.3.4. Physics-of-Failure Analysis Prediction Method

The objective of physics-of-failure analysis prediction method is to determine or predict failures when a specific end-of-life failure mechanism will occur for an individual component in a specific application. This method requires detailed knowledge of all material characteristics, geometries, and environmental conditions.

3.3.4. Reliability Analysis Techniques

There are several reliability analysis techniques in the literature. In **Table 4**, some of the frequently used reliability analysis techniques are given with their purposes as used in the literature [27].

Table 4: Reliability analysis techniques and their purposes

Analysis	Purpose
Dormancy Analysis	Dormancy Analysis is used to calculate failure rates of devices during dormant (e.g., storage). Determination of the effects of expected periods of storage or other non-operating conditions on the reliability of the product can be obtained.
Durability Assessment	Durability Assessment is used to confirm a design life for a product, such as determination of whether or not the mechanical strength of a product will remain adequate for its expected life. It is more effectively applied earlier in development to ensure that design life is adequate.
Failure Modes, Effects, and Criticality Analysis	FMECA is used ideally as a design and assessment tool to understand and alleviate failure consequences, it can also be independently applied tool to avoid certain failure consequences. It provides a qualitative measurement.
Fault Tree Analysis (FTA)	FTA is used ideally as a design and assessment tool to understand and alleviate failure consequences using inductive logic to determine the possible causes of an undesired operational result. It can also be an independently applied tool

	to check that certain failure consequences are avoided.
Finite Element Analysis (FEA)	FEA is a computer simulation technique used for predicting material response or behavior of modeled device by decomposing the product into simple elements, determining material stresses and temperature, and determining thermal and dynamic loading.
Sneak Circuit Analysis (SCA)	SCA is used ideally as a design and assessment tool to discover unintended paths and functions in a product, it can also be an independently applied tool to check that certain failure consequences are avoided.
Thermal Analysis (TA)	TA is used to analyze of the heat dissipations, transfer paths and cooling sources to determine if part/product temperatures are consistent with reliability needs.
Worst Case Circuit Analysis (WCCA)	WCCA is a tool used to effectively assess design tolerance to parameter variation in the components of a product, it can also be used as an independent check of the susceptibility to variation.

3.3.4.1. Dormancy Analysis

The purpose of performing a *dormancy analysis* is to assess the effects of environmental storage parameters on product characteristics such as performance, lubrication. A well-defined analysis can isolate problem areas that should be candidates for design or process change [27].

Applying analysis early to a product that will experience extended non-operating conditions will result in lower product life cycle costs, higher product reliability, reduction of experienced failure mechanisms and ultimate customer satisfaction. For those times when long storage periods are expected, a dormancy analysis can determine if periodic testing is necessary to ensure proper operation.

3.3.4.2. Durability Analysis

The primary purpose of a *durability analysis* is to identify components and processes that exhibit "early" wearout failure, isolate the root cause and determine potential corrective actions. Durability analysis is an analysis technique that focuses on identifying and solving design problems related to early product or materials wearout [27].

The benefits of an effective durability analysis are fewer failures experienced during the useful life of the product and greater customer satisfaction for the product. For the design team, the durability analysis provides detailed analytical models that assess the physical relationships between the product application and the operating environment.

3.3.4.3. Fault Tree Analysis (FTA)

Fault Tree Analysis (FTA) is a top down failure consequence assessment technique that is useful in identifying safety concerns so that product modifications can be made. When used in the design stage, the results of the analysis can identify the cause(s) of product failures which may then be eliminated through good design practice. Updating the FTA to reflect design changes can assess whether previous problems have been eliminated, or new problems have been introduced [27].

When FTA is applied in the design stage, the benefits that can be derived include:

- Identification of single failure points
- Identification of safety concerns
- Evaluation of software and man-machine interfaces
- Evaluation of design change impacts
- Simplification of maintenance and trouble-shooting procedures
- Assessment of modifications or enhancements

An FTA can be performed as early as the product Concept/Planning phase; however, application in the early stages of Design/Development is the most informative. This technique is effective for assessing design progress in identifying the causes of failure in a product resulting from modifications and design corrective actions [27].

As a general assessment tool, FTA can be used for evaluation of complex products with regard to safety and reliability. This technique can be applied when the need to know the causes of a hypothesized catastrophic event is important for the success of a product. Similar to a FMEA, FTA can identify major failure modes of the product resulting from lower level failures. The product design reliability can then be improved by eliminating the causes of those failures [27].

3.3.4.4. Finite Element Analysis (FEA)

Simulation techniques are very effective checks of mechanical and thermal robustness of product designs prior to production. *Finite Element Analysis (FEA)* is a simulation technique, usually computer implemented, that estimates material response to loads or environmental disturbances [27]. The analysis can be used to assess the potential for thermal or mechanical failure in reaction to the expected loads, or assessment of failures resulting from testing.

Application of an FEA is especially appropriate for products that use advanced or unique packaging or design concepts. The types of problems that can be analyzed include mechanical stress analysis, heat transfer, fluid flow, vibration and elasticity [27].

The benefits of an FEA are the early discovery of life limiting material deficiencies and the uncovering of excessive environmental load conditions. With the identification of the deficiency, either more robust components or better environment isolation techniques can be introduced to reduce the load's impact on the product design. This analysis can be performed before product manufacturing to uncover problems, after design changes to detect weaknesses, or after determining problematic areas through testing [27].

The most effective FEA occurs when the product or item is developed to the point where the material and design properties can be clearly defined. Since FEA is time consuming and costly, the items to be analyzed should be selected very carefully. When used as an assessment tool, failure trends or problematic areas would be potential candidates for the analysis [27].

3.3.4.5. Sneak Circuit Analysis

The purpose of performing a S*neak Circuit Analysis (SCA)* is to find and fix each sneak failure cause to improve the product design [27]. Iterative sneak analysis can assess progress in identifying and eliminating problems that may unexpectedly occur during normal product operation or through incorporation of design modifications and "fixes". The only preventive measure for identifying these sneak circuits is an in-depth manual or computer-aided circuit analysis.

Finding and correcting design flaws before using a product can enhance customer satisfaction. In addition, reassessments should be performed every time a design change is occurred. With the development of automated tools, all computer-aided designs can be checked almost as easily as a text document can be spell-checked. These tools increase the scope of application significantly. Specific benefits include [27]:

- Detection of hidden failures
- Prevention of costly redesigns
- Verification of circuit interface integrity
- Ensuring high reliability
- Avoidance of litigation resulting from undiscovered sneak paths

3.3.4.6. Thermal Analysis

The general purpose of a *Thermal Analysis (TA)* is to determine the adequacy of the thermal design of the product in its intended use environment [27]. This analysis is a relatively cost effective technique to assess thermal characteristics from the component to the product level.

TA is a useful technique in characterizing product temperature profiles without resorting to testing. The most important benefits are:

- Estimating operating temperatures of parts and components to assess compliance with customer needs/requirements
- Determining thermal expansion of materials to aid in material selection based on their coefficient of expansion
- Identifying hot spot areas or parts exceeding allowable limits for design/part selection trade-offs
- Optimizing the product thermal design to maximize inherent reliability

Evaluation of the thermal design starts during the Concept/Planning phase and is an on-going assessment. Each change or modification to the product design requires a re-evaluation to ensure that the gross thermal conditions are controlled for the expected environment and type of product [27].

3.3.4.7. Worst Case Circuit Analysis (WCCA)

A *Worst Case Circuit Analysis (WCCA)* technique is used to assess progress in desensitizing the product to extreme environmental conditions or component parameter changes as the product is used [27]. The main benefits associated with conducting a worst case circuit analysis are:

- Identifies parts exceeding component derating limit guidelines/requirements
- Analyzes circuits for design faults
- Identifies components that may be overstressed

- Provides a realistic estimate of true worst case performance
- Provides information on possible life limiting conditions and components
- Exposes failures that may be safety risks

Due to the need for detailed information on the design, materials, parts and processes, this analysis technique is not recommended for application during the product Concept/Planning phase. The best time to use it, would be after the initial design review (early Design/Development), with appropriate updates as the product design changes to determine critical component parameter variation and environmental effects on circuit performance. The further along in the design and development phase that WCCA is performed, the more expensive it will be to introduce changes to the design [27].

3.3.4.8. Failure Modes and Effects Analysis (FMEA)

This technique was first used in the development of flight control systems in the early 1950s. FMEA method is concerned with determining design reliability by considering potential failures and their effect on the system under study. FMEA and FMECA is a formalized design process with an objective to improve the inherent reliability. This is an iterative process that influences design by identifying failure modes, assessing their probabilities of occurrence and their effects on the system. It may also consider isolation of the causes and determining corrective actions or preventive measures [27], [35].

3.3.4.8.1. Steps of FMEA

FMEA study comprises of following steps:

3.3.4.8.1.1. System definition

This step is to identify those system components that will be subject to failure. A functional and physical description of the system provides the definition and boundaries for performing analysis.

3.3.4.8.1.2. Identification of failure modes

Failure modes will be identified by hardware or function approach. Failures modes are observable manners in which a component fails. For example: valve open, circuit short, pipe or valve rupture, power loss, etc.

3.3.4.8.1.3. Determination of causes

For each failure mode an assessment is made as to the probable cause or causes. A failure mode may have more than one cause.

3.3.4.8.1.4. Effect assessment

The impact each failure has on the operation or status of the system is assessed. Effects may range from complete system failure to partial degradation to no impact on performance.

3.3.4.8.1.5. Classification of severity

A severity classification is assigned to each failure mode to be used for prioritization of corrective actions. Generally severity is classified in four classes [32], [34].

Category I : Catastrophic. Significant system failure occurs that can result in injury, loss of life, or major damage.

Category II : Critical. Complete loss of system occurs, performance unacceptable.

Category III : Marginal. System is degraded with partial loss in performance.

Category IV : Negligible. Minor failure occurs with no effect on acceptable system performance

3.3.4.8.1.6. Estimation of probability of occurrence

Probability of occurrence of each failure mode is estimated generally using handbook, existing databases or expert opinions. Some of the standard handbook on FMEA classifies qualitatively frequency of occurrence in five major levels [32], [34]:

Level A : Frequent: High probability of failure ($p \geq 0.20$)

Level B : Probable: Moderate probability of failure ($0.1 \leq p < 0.20$)

Level C : Occasional: Marginal probability of failure ($0.01 \leq p < 0.1$)

Level D : Remote: Unlikely probability of failure ($0.001 \leq p < 0.01$)

Level E : Extremely unlikely: Rare probability of failure ($p < 0.001$)

3.3.4.8.1.7. Computation of criticality index

This is a quantitative measure of the criticality of the failure mode that combines the probability of the failure mode's occurrence with its severity ranking [34]. The index may be defined as:

$$C_k = \alpha_{kp}\beta_k\lambda_p t \qquad (28)$$

where C_k is critical index for failure mode k

α_{kp} the fraction of the component p's failure having failure mode k

β_k the conditional probability that failure mode k will result in the identified failure effect

λ_p the failure rate of component p

t duration of time used in the analysis

β_k, the conditional probability is a subjective estimate that may be quantified as given in **Table 5** [34].

Table 5: Categories of Failure Effects

Failure effect	β
Certain	$\beta=1.0$
Probable	$0.10<\beta<1.0$
Possible	$0<\beta<0.10$
No effect	$\beta=0$

For a given p, the sum of α_{kp} over all its failure modes would normally equal 1. Failure mode classification may be performed in accordance with **Table 6** [34]. Corrective actions are to be performed according to the result of classification.

Table 6: Failure mode classification matrix

Criticality index	*Severity classification*				
	A	IV	III	II	I
	B				
	C				
	D				
	E				

FTA and FMEA are based on the failures of the system and they are systematic methods to define general system failures and their conditions. The rest of the methods mentioned above have specific application areas such as thermal effects, material characterizations, environmental conditions etc [34].

Since the design is overviewed by considering all the failure modes and evaluates the probabilities and effects of the events on the system, FMEA is an applicable tool to interfere expert opinions for reliability evaluation of the system. FMEA is also based on expert opinions. For this reason, FMEA is selected as a method to construct fuzzy rule base in this thesis and used as an evaluation method for different probabilities and effects.

CHAPTER 4

FUZZY LOGIC AND RELIABILITY

4.1. Fuzzy Sets and Fuzzy Logic

Conventional logic system defines the result of an event with a definite statement. Fuzzy Logic was introduced by Lotfi A. Zadeh in 1965 [29] as a mathematical way to rationalize uncertain events. Fuzzy Logic deals with uncertainty and imprecision in engineering by attaching degrees of certainty to the answer of a logical question, thus allows intermediate values to be defined between conventional evaluations.

Human decisions generally do not include certain statements; they can be categorized in several uncertain categories. Fuzzy Logic imitates the human reasoning by using levels of possibility in a number of uncertain (fuzzy) categories.

Some examples to indicate for the difference between conventional and fuzzy logic are given in **Table 7** [23], [24]. As it can be seen from **Table 7**, fuzzy logic contains categorized uncertain parameters rather than certain statements, which is same as human reasoning.

Table 7: Difference between Conventional and Fuzzy Logic

Conventional Logic	**Fuzzy Logic**
	
0 / 1	(0,1)
Yes / No	Definitely yes, Probably yes, Maybe Probably no, Definitely no
High / Low	High / Moderate / Low

The most important features of the Fuzzy Logic are the possibility to express the system knowledge by means of linguistic expressions rather than mathematical equations and to deal with such a kind of uncertainty and imprecision. Brief chronology of the most important fuzzy logic applications are listed in **Table 8** [19];

Table 8: Chronological Order of Fuzzy Applications

Year	**Application**
1965	Concept of fuzzy sets theory by Lotfi Zadeh (USA)
1972	First working group on fuzzy systems in Japan by Toshiro Terano
1973	Paper about fuzzy algorithms for complex systems and decision processes by Zadeh (USA)
1974	Steam engine control by Ebrahim Mamdani (UK)
1977	First fuzzy expert system for loan applicant evaluation by Hans Zimmermann (Germany)
1980	Cement kiln control by F. – L. Smidth & Co. – Lauritz P. Holmblad (Denmark) – the first permanent industrial application Fuzzy logic chess and backgammon program – Hans Berliner (USA)
1984	Water treatment (chemical injection) control (Japan)

	Subway Sendai Transportation system control (Japan)
1985	First fuzzy chip developed by Masaki Togai and Hiroyuke Watanabe in Bell Labs (USA)
1986	Fuzzy expert system for diagnosing illnesses in Omron (Japan)
1987	Container crank control Tunnel excavation Soldering robot Automated aircraft vehicle landing Second IFSA Conference in Tokyo Togai InfraLogic Inc. – first fuzzy company in Irvine (USA)
1988	Kiln control by Yokogawa First dedicated fuzzy controller sold – Omron (Japan)
1989	Creation of Laboratory for International Fuzzy Engineering Research (LIFE) in Japan
1990	Fuzzy TV set by Sony (Japan) Fuzzy electronic eye by Fujitsu (Japan) Fuzzy Logic Systems Institute (FLSI) by Takeshi Yamakawa (Japan) Intelligent Systems Control Laboratory in Siemens (Germany)
1991	Fuzzy AI Promotion Centre (Japan) Educational kit by Motorola (USA)

In the following years, many applications in a wide domain are available and hundreds of studies on fuzzy logic have been performed up to now.

Typical Fuzzy Logic System is mainly composed of the elements such as fuzzy rule base, membership functions, fuzzification, fuzzy inference process and defuzzification. The relation between these elements is shown in **Figure 10** [36] for typical fuzzy logic systems.

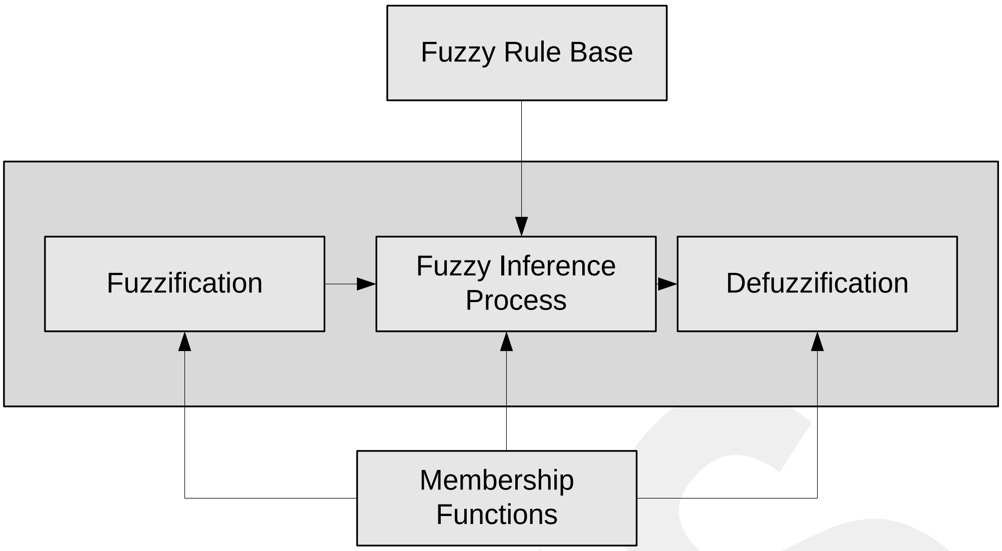


Figure 10: Fuzzy Logic System

4.2. Introduction to Fuzzy Logic

4.2.1. Fuzzy Sets

Fuzzy set is a set without a crisp, clearly defined boundary. Let U be a collection of objects, for example $U = R^n$, and be called the universe of discourse. A fuzzy set A in U is characterized by a membership function $\mu_A : U \rightarrow [0,1]$, with representing the grade of membership of $u \in U$ in the fuzzy set A [25]. A fuzzy set may be viewed as a generalization of the concept of a crisp set, whose membership function only takes two values {0,1}.

4.2.2. Logical Operations

In fuzzy logic reasoning covers the Boolean logic, since it uses the extreme values for fuzzy variables. For “A AND B” statement, where A and B are can be real numbers in the range of (0,1), statement is resolved as min (A, B). “AND” logical operator is explained in **Table 9**.

Table 9: AND Logical Operation

A	B	A AND B $\min(A, B)$
0	0	0
0	1	0
1	0	0
1	1	1

For "A OR B" statement, where A and B are can be real numbers in the range of (0,1), statement is resolved as $\max(A, B)$. "OR" logical operator is explained in **Table 10**.

Table 10: OR Logical Operation

A	B	A OR B $\max(A, B)$
0	0	0
0	1	1
1	0	1
1	1	1

4.2.3. Operations on Fuzzy Sets

The relationships of fuzzy subsets A and B of $X = R^n$ having the membership values and for $x \epsilon X$ are given below [25];

I. A is equal to B, $A = B$

$$\mu_A(x) = \mu_B(x) \text{ for all } x \epsilon X$$

II. A is complement of B, $A = \bar{B}$

$$\mu_A(x) = \mu_{\bar{B}}(x) = 1 - \mu_B(x) \text{ for all } x \epsilon X$$

III. The union of A and B, $A \cup B$

$$\mu_{A\cup B}(x) = \max\{\mu_A(x), \mu_B(x)\} \quad \text{for all } x \in X$$

IV. The intersection of A and B, $A \cap B$

$$\mu_{A\cap B}(x) = \min\{\mu_A(x), \mu_B(x)\} \quad \text{for all } x \in X$$

V. The complement $\bar{A}$ of A is;

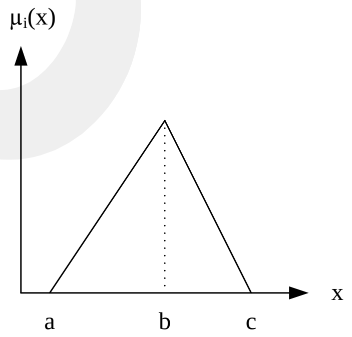
$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$

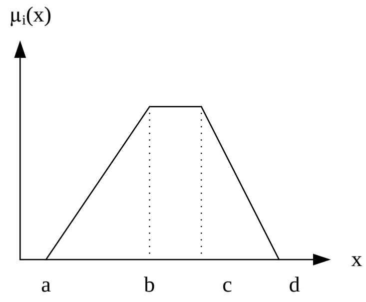
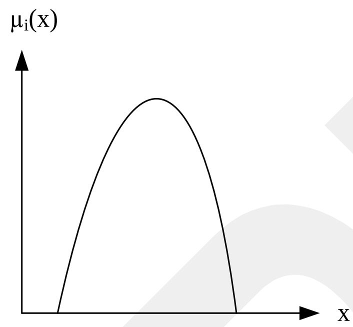
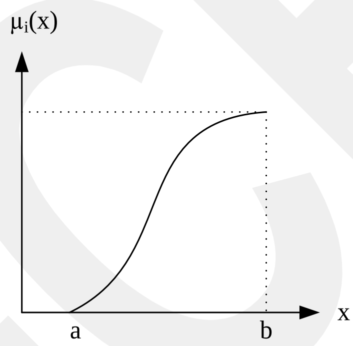
4.2.4. Fuzzy Rule Base

The relationship between input and output variables are described in a rule base, composed of IF… THEN form rules.

The fuzzy rule base is the heart of the fuzzy logic system to be used in a specific problem. If the rules are determined by numerical data, then the first step in fuzzy rule base is to determine the functional forms of the membership functions. The most commonly used functional forms are Trapezoid, Triangular, Gaussian, the sigmoid curve, quadratic and cubic polynomial curves, as given in **Table 11** [23].

Table 11: The Most Commonly Used Functional Forms

Form	Membership Function	Functions
Triangular		$\mu_A(x;a,b,c) = \begin{cases} 0 & for\ \ x < a \\ \left(\frac{x-a}{b-a}\right) & for\ a \le x \le b \\ \left(\frac{c-x}{c-b}\right) & for\ b \le x \le c \\ 0 & for\ \ x > c \end{cases}$

Trapezoid	 	$$\mu_A(x;a,b,c,d) = \begin{cases} 0 & for\ \ x < a \\ \left(\frac{x-a}{b-a}\right) & for\ a \le x \le b \\ 1 & for\ b \le x \le c \\ \left(\frac{c-x}{c-b}\right) & for\ c \le x \le d \\ 0 & for\ \ x > d \end{cases}$$
Gaussian	 	$$\mu_A(x;m,\sigma) = exp\left\{\frac{-(x-m)^2}{2\sigma^2}\right\}$$
Sigmoid	 	$$\mu_A(x;a,b) = \left\{\frac{1}{1+e^{-a(x-b)}}\right\}$$

4.2.5. Linguistic Variables

The concept of linguistic variables was introduced by Zadeh (1973) to provide a basis for approximate reasoning. If a variable can take words in natural languages as its values, this type of variable is defined as a linguistic variable. Linguistic variables play the same role as the numerical variables in conventional mathematical models, but take on words or sentences in a natural language, i.e., “definitely yes”, “probable yes”, “maybe” as their values. The value of linguistic variable is assigned with appropriate fuzzy set [18].

4.2.6. Fuzzy Inference Rules

A fuzzy rule base consists of a collection of fuzzy IF-THEN rules. The most commonly used fuzzy inference rules are Generalized Modus Ponens (GMP) and Generalized Modus Tollens (GMT).

4.2.6.1. Generalized Modus Ponens (GMP)

GMP is the direct reasoning. GMP is an inference mechanism that allows obtaining *imprecise* conclusion from *imprecise* (vague) fact. GMP is defined symbolically below.

Let A and B fuzzy subsets on the sets X and Y, the fuzzy conditional statement [25], [26];

Implication : If x is A, then y is B
Premise : x is A’
Conclusion : y is B’.

4.2.6.2. Generalized Modus Tollens (GMT)

GMT can be explained as given below;

Implication : If x is A, then y is B
Premise : y is not B’
Conclusion : x is not A’.

4.2.6.3. Generalized Hypothetical Syllogism (GHS)

GHS can be explained as given below;

Implication	: If x is A, then y is B
Premise	: If y is B' then z is C
Conclusion	: z is C'.

4.2.7. Fuzzification

Fuzzification process is a mathematical procedure for converting an element in the universe of discourse into the membership value of the fuzzy set. There are 2 (two) fuzzification methods; Fuzzy Singleton and Fuzzy Non-singleton. If the input is a numerical value and noise free, then the fuzzification operator is singleton. Non-singleton fuzzification method is used when the input is noisy and it is based on a fuzzy set and the input is generally the center of the set [23]. Singleton and non-singleton fuzzification methods are shown in **Figure 11**.

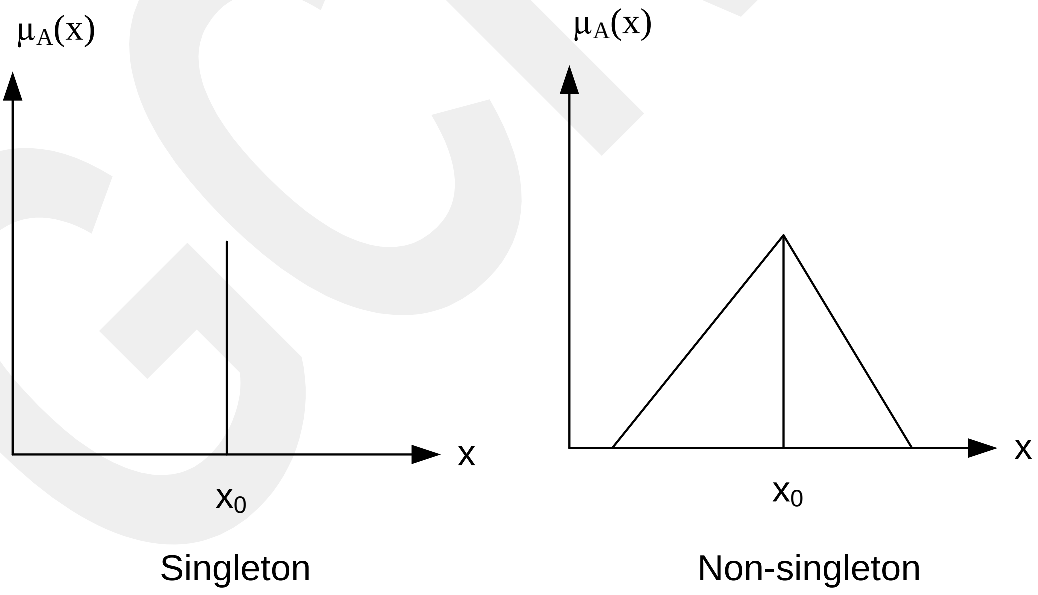


Figure 11: Singleton and Non-singleton Fuzzification Methods

4.2.8. Fuzzy Inference Engine

Fuzzy Inference mechanism calculates the degree to which each rule fires for a given fuzzified input by considering the rules. A rule fires when the degree of membership of the IF part is higher than 0.

In a fuzzy inference engine, fuzzy logic principles are used to combine fuzzy rules in the fuzzy rule base. Fuzzy inference is the process of formulating the mapping from a given input to an output using fuzzy logic. The process of fuzzy inference involves all of the elements: membership functions, fuzzy logic operators, and if-then rules. The most commonly used fuzzy inference systems are Mamdani-type and Sugeno-type (also known as the TSK fuzzy model).

Mamdani-type fuzzy inference method was proposed in 1975 by Ebrahim Mamdani as an attempt to control a steam engine and boiler combination by synthesizing a set of linguistic control rules obtained from experienced human operators which was based on Lotfi Zadeh's paper in 1973 on fuzzy algorithms for complex systems and decision processes [19]. Mamdani-type inference expects the output membership functions to be fuzzy sets. Mamdani-type fuzzy inference method is the most commonly seen fuzzy methodology.

Sugeno-type inference system was introduced in 1984 by Takagi, Sugeno and Kang. Sugeno-type systems can be used to model any inference system in which the output membership functions are either linear or constant [19]. For complex and high-dimensional problems, Sugeno-type inference is used to develop a systematic approach to generate fuzzy rules from a given input/output data set. However Sugeno-type model replaces the fuzzy sets of Mamdani rule with functions (equations) of the input variables [20]. Comparison of these two types of inference is given in **Table 12** [21].

Table 12: Comparison of Sugeno and Mamdani Type Inference

Advantages of Sugeno-type inference	**Advantages of Mamdani-type inference**
computationally efficient	Intuitive
works well with linear techniques	widespread acceptance
works well with optimization and adaptive techniques	well-suited to human input
guaranteed continuity of the output surface	
well-suited to mathematical analysis	

Sugeno fuzzy inference system has difficulties in dealing with the multi-parameter synthetic evaluation; it has difficulties in assigning weight to each input and fuzzy rules. Mamdani model can show its legibility and understandability to the laypeople. The Mamdani fuzzy inference system shows its advantage in output expression [21]. However, Sugeno fuzzy inference system provides a possibility of numerical classification of the output.

4.2.9. Defuzzification

Defuzzification process compiles the information provided by each of the rules and makes a decision from this basis. Defuzzification is the conversion of fuzzy output to precise (crisp) value. The input for the defuzzification is a fuzzy set and the output is a single crisp number.

There are different methods for the calculation of crisp output of fuzzy system like bisector, centroid, middle of maximum (MOM), smallest of maximum (SOM) and largest of maximum (LOM).

Centroid is the most frequently used methods for defuzzification. Centroid defuzzified output is calculated as weighted average over the whole universe of output. For illustration, all of the above mentioned defuzzification method results are given in **Figure 12** [37]. For different applications, different defuzzification methods can be selected for use.

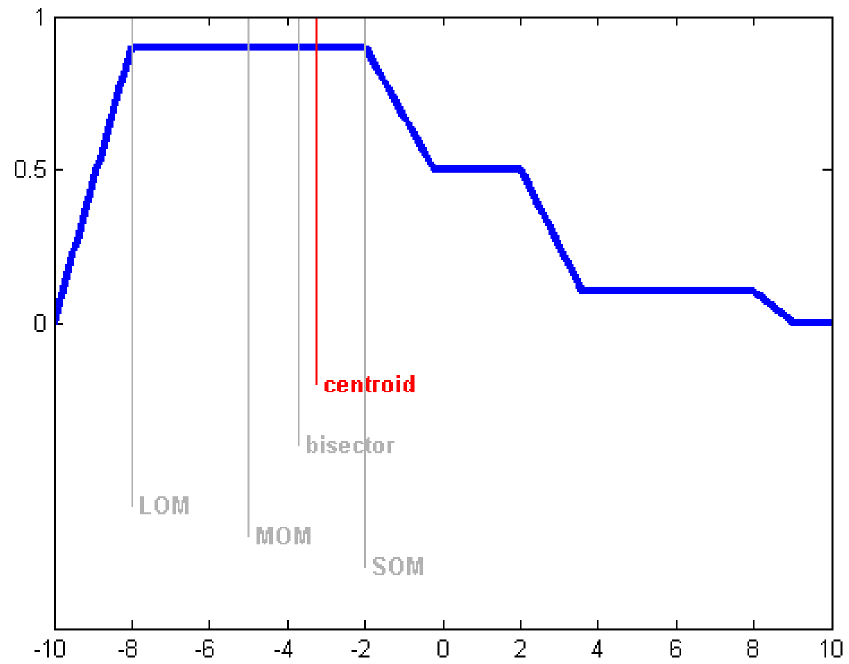


Figure 12: Defuzzification methods

4.3. Adaptive Neuro-Fuzzy Inference Systems (ANFIS)

ANFIS is an adaptive network that is functionally equivalent to fuzzy inference system [28]. An adaptive network is network of nodes and directional links. Since it integrates both neural networks and fuzzy logic principles, it has potential to capture the benefits of both in a single framework such as representation of prior knowledge into a set of constraints and adaptation of back-propagation to structured network.

In ANFIS, Takagi-Sugeno type fuzzy inference system is used [28]. The output of each rule can be a linear combination of input variables plus a constant term or can be only a constant term. The final output is the weighted average of each rule's output.

The ANFIS approach learns the rules and membership functions from data. These networks are learning a relationship between inputs and outputs. ANFIS structure is represented in **Figure 13**.

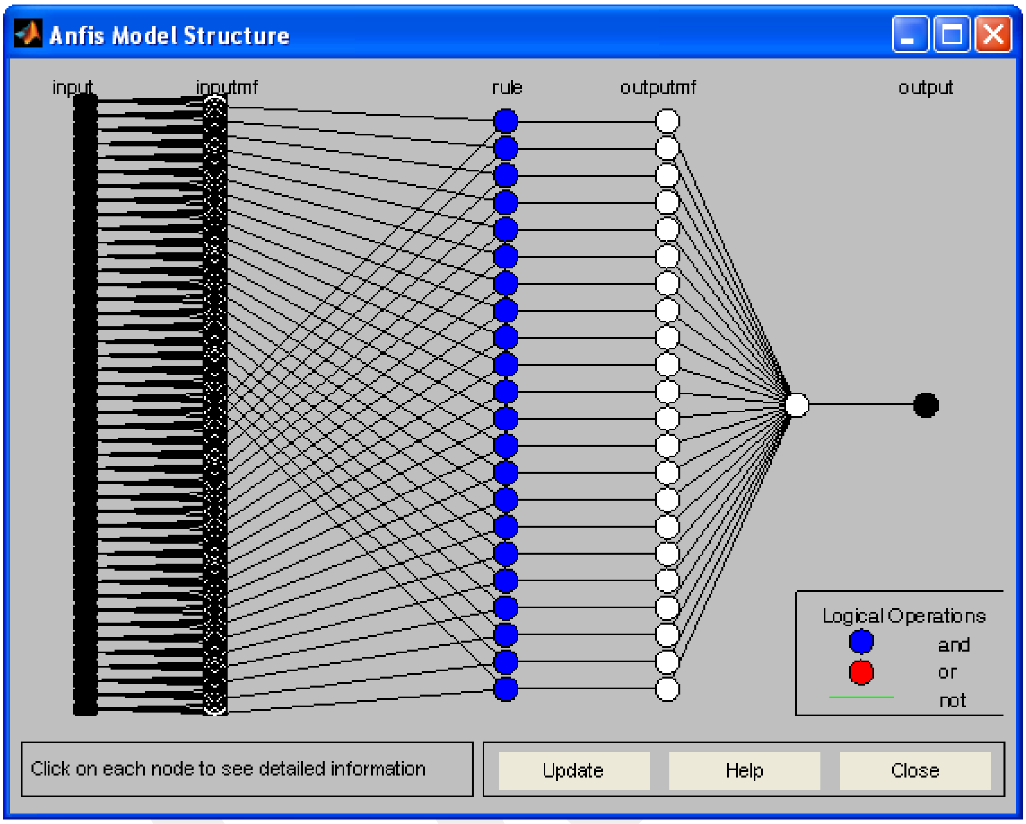


Figure 13: ANFIS Structure

For Sugeno type inference systems, ANFIS can be used to define the membership functions and fuzzy logic structure.

4.4. Fuzzy Logic for Missile Reliability

4.4.1. Fuzzy Logic System Description for This Thesis

The Fuzzy Logic System in this thesis can be summarized as shown in **Figure 14**. The relations and data obtained in FMEA study are used to construct the fuzzy rule base. The assessed expert opinions including the effects on the system and probability of occurrence values are used as the inputs to the fuzzification process. Results are categorized by the experts.

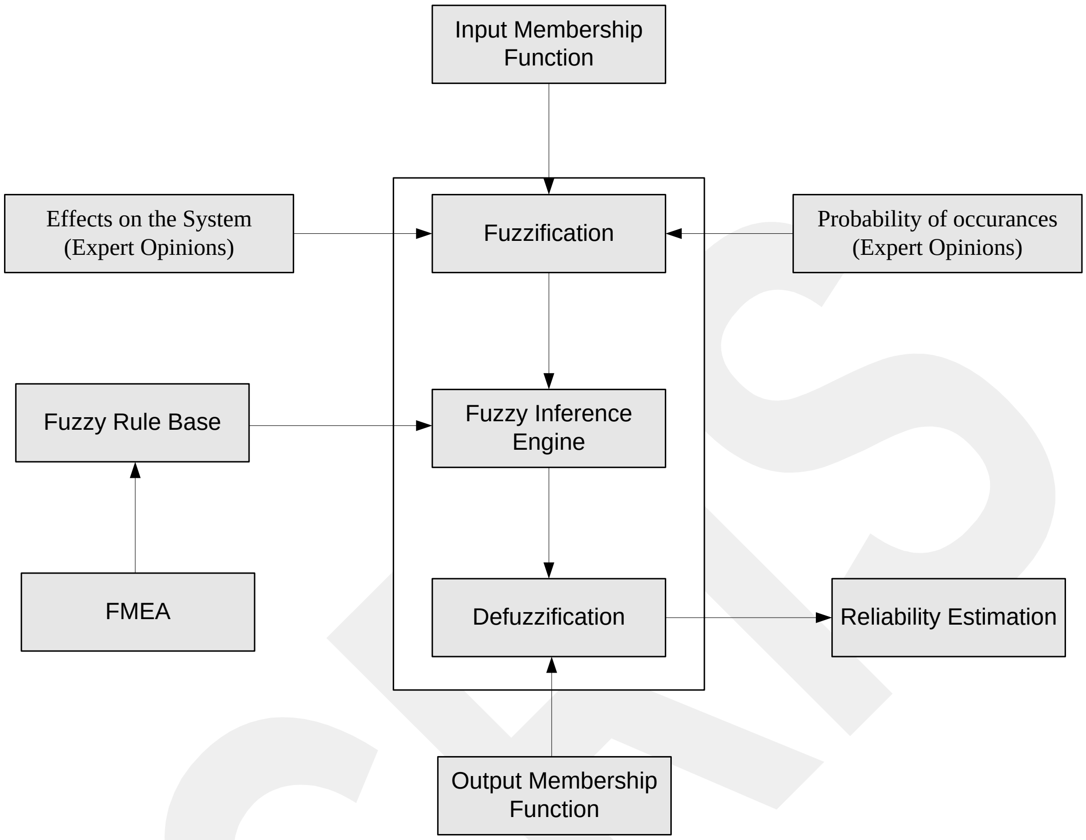


Figure 14: Fuzzy Logic System

The relationship between input and output variables are described in a rule base, composed of IF… THEN form rules by using the FMEA study results.

In this thesis, membership functions are constructed according to the categories of effects on system and probability of occurrences separately in accordance with the relations obtained in FMEA study. Linguistic rules are defined by using linguistic variables which play the same role as the numerical variables in conventional mathematical models, but take on words or sentences in a natural language, i.e., “definitely yes”, “probable yes”, “maybe” as their values. The value of linguistic variable is assigned within an appropriate fuzzy set.

Five experts are asked to assign values between [0-100] for probability of occurrence and effect on system for the items in FMEA. Criticality index computation for the items in FMEA can be performed if the size of the model is too large. The categorization process of expert opinions is handled as fuzzy variables according to the definitions given below in **Table 13** and **Table 14**.

Values in **Table 13** and **Table 14** are assigned as a result of discussions with experts on how to categorize the opinions and system performance. The means of the assigned values by the experts are taken and used in the fuzzy system.

Table 13: Categorization of the Expert Opinions for Probability of Occurrence

Range	**Category of the Probability of Occurrence**	**Category of the Effect on System**
0-30	Category-E Rare: Occurrence of such a failure is too rare and not expected in normal conditions.	Category-5: Negligible
20-50	Category-D Remote: Remote to occur in single item’s life cycle, however can be observed in other items but to be considered in case of occurrence.	Category-4: Marginal
40-70	Category-C Occasional: Forecasted occurrence of once a time in systems life cycle.	Category-3:Critical
60-90	Category-B Probable: Forecasted occurrence of couple of times during the systems life cycle.	Category-2: Moderate
80-100	Category-A Frequently: Forecasted as frequently observable failures.	Category-1: Catastrophic

Table 14: Categorization of the Expert Opinions for the Reliability Evaluation

Range	Category of the Probability for Working Successfully
0-60	Failure
60-80	Moderate
80-100	Success

Both types of the inference systems named as Sugeno and Mamdani, can be used in the fuzzy system. If the result can be stated with an exact value, then Sugeno type inference system can be used. If the result is composed of different categories, which can't be stated with a single value, then Mamdani type inference system can be applied.

The proposed method is simple applied on the following example. A failure for a hypothetical system is defined in **Table 15**.

Table 15: Hypothetical System Failure

Failure Mode	Probability of Occurrence (0-100)	Category of the Probability of Occurrence	Severity (0-100)	Category of the Severity	Contribution on System Reliability (0-100)
Event	86	A & B	82	1 & 2	F

Let two rules defined according to **Table 15** as follows;

If (P = A) & (E = 1) then R = Failure

If (P = B) & (E = 2) then R = Moderate.

The input membership functions are defined for both probability of occurrence and effect on system as given in Eq.(32) to Eq.(36) and Eq.(37) to Eq.(41) respectively, in accordance with **Table 13**.

Input membership function of probability of occurrence for category “E”;

$$\mu_E(x;a,b,c,d)=\left\{\begin{array}{cc} 0 & for\ \ 30<x\leq 100 \\ \left(\dfrac{30-x}{15}\right) & for\ 15\leq x\leq 30 \\ 1 & for\ \ 0\leq x<15 \end{array}\right\} \qquad (32)$$

Input membership function of probability of occurrence for category “D”;

$$\mu_D(x;a,b,c)=\left\{\begin{array}{cc} 0 & for\ \ x<20 \\ \left(\dfrac{x-20}{15}\right) & for\ 20\leq x\leq 35 \\ \left(\dfrac{50-x}{15}\right) & for\ 35\leq x\leq 50 \\ 0 & for\ \ x\geq 50 \end{array}\right\} \qquad (33)$$

Input membership function of probability of occurrence for category “C”;

$$\mu_C(x;a,b,c)=\left\{\begin{array}{cc} 0 & for\ \ x<40 \\ \left(\dfrac{x-40}{15}\right) & for\ 40\leq x\leq 55 \\ \left(\dfrac{70-x}{15}\right) & for\ 55\leq x\leq 70 \\ 0 & for\ \ x\geq 70 \end{array}\right\} \qquad (34)$$

Input membership function of probability of occurrence for category “B”;

$$\mu_B(x;a,b,c)=\left\{\begin{array}{cc} 0 & for\ \ x<60 \\ \left(\dfrac{x-60}{15}\right) & for\ 60\leq x\leq 75 \\ \left(\dfrac{90-x}{15}\right) & for\ 75\leq x\leq 90 \\ 0 & for\ \ x\geq 90 \end{array}\right\} \qquad (35)$$

Input membership function of probability of occurrence for category “A”;

$$\mu_A(x; a, b, c, d) = \begin{cases} 0 & for \;\; x < 80 \\ \left(\frac{x - 80}{10}\right) & for \; 80 \le x \le 90 \\ 1 & for \; 90 < x \le 100 \end{cases} \qquad (36)$$

Input membership function of effect on system for category "5";

$$\mu_5(x; a, b, c, d) = \begin{cases} 0 & for \;\; 30 < x \le 100 \\ \left(\frac{30 - x}{15}\right) & for \; 15 \le x \le 30 \\ 1 & for \;\; 0 \le x < 15 \end{cases} \qquad (37)$$

Input membership function of effect on system for category "4";

$$\mu_4(x; a, b, c) = \begin{cases} 0 & for \;\; x < 20 \\ \left(\frac{x - 20}{15}\right) & for \; 20 \le x \le 35 \\ \left(\frac{50 - x}{15}\right) & for \; 35 \le x \le 50 \\ 0 & for \;\; x \ge 50 \end{cases} \qquad (38)$$

Input membership function of effect on system for category "3";

$$\mu_3(x; a, b, c) = \begin{cases} 0 & for \;\; x < 40 \\ \left(\frac{x - 40}{15}\right) & for \; 40 \le x \le 55 \\ \left(\frac{70 - x}{15}\right) & for \; 55 \le x \le 70 \\ 0 & for \;\; x \ge 70 \end{cases} \qquad (39)$$

Input membership function of effect on system for category "2";

$$\mu_2(x; a, b, c) = \begin{cases} 0 & for \;\; x < 60 \\ \left(\frac{x - 60}{15}\right) & for \; 60 \le x \le 75 \\ \left(\frac{90 - x}{15}\right) & for \; 75 \le x \le 90 \\ 0 & for \;\; x \ge 90 \end{cases} \qquad (40)$$

Input membership function of effect on system for category "1";

$$\mu_1(x;a,b,c,d)=\begin{Bmatrix} 0 & for \ \ x<80 \\ \left(\frac{x-80}{10}\right) & for\ 80\le x\le 90 \\ 1 & for\ 90<x\le 100 \end{Bmatrix} \quad (41)$$

Output membership functions for reliability evaluation are defined in Eq.(42) to Eq.(44). in accordance with **Table 14**.

Output membership function for category "Failure";

$$\mu_{Failure}(x;a,b,c,d)=\begin{Bmatrix} 0 & for \ \ 60<x\le 100 \\ \left(\frac{60-x}{15}\right) & for\ 45\le x\le 60 \\ 1 & for\ 0\le x<60 \end{Bmatrix} \quad (42)$$

Output membership function for category "Moderate";

$$\mu_{Moderate}(x;a,b,c)=\begin{Bmatrix} 0 & for \ \ x<50 \\ \left(\frac{x-50}{20}\right) & for\ 50\le x\le 70 \\ \left(\frac{90-x}{20}\right) & for\ 70\le x\le 90 \\ 0 & for\ \ x\ge 90 \end{Bmatrix} \quad (43)$$

Output membership function for category "Success";

$$\mu_{Success}(x;a,b,c,d)=\begin{Bmatrix} 0 & for \ \ x<80 \\ \left(\frac{x-80}{10}\right) & for\ 80\le x\le 90 \\ 1 & for\ 90<x\le 100 \end{Bmatrix} \quad (44)$$

If the model is applied for P=86 and E=82;

- P = 86 ⇒

  For "A" membership;

$$P(A) = \frac{86 - 80}{10} = \frac{6}{10}$$

For “B” membership;

$$P(B) = \frac{90 - 86}{15} = \frac{4}{15}$$

- E = 82⇒

For “1” membership;

$$P(1) = \frac{82 - 80}{10} = \frac{2}{10}$$

For “2” membership;

$$P(2) = \frac{90 - 82}{15} = \frac{8}{15}$$

The obtained *P(x)* values are used in the fuzzy rule as follows;

- If (P = A) & (E = 1) then R = **Failure**

  $0.6 \wedge 0.2 = 0.2$

  ⇒ **Result = 0.2**

- If (P = B) & (E = 2) then R = **Moderate**

  $0.266 \wedge 0.533 = 0.266$

  ⇒ **Result = 0.266**

It can be stated that Result is 20% Failure and 26.66% Moderate according to the calculation above. If CoG is selected as a defuzzification method, followings are obtained;

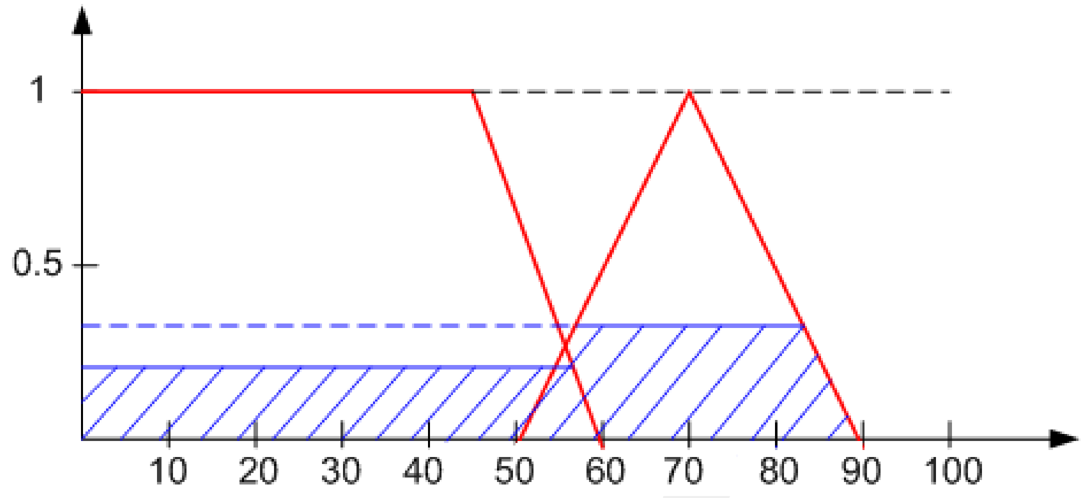


Figure 14: Defuzzification

Obtained CoG in x-axis is shown in **Figure 16**.

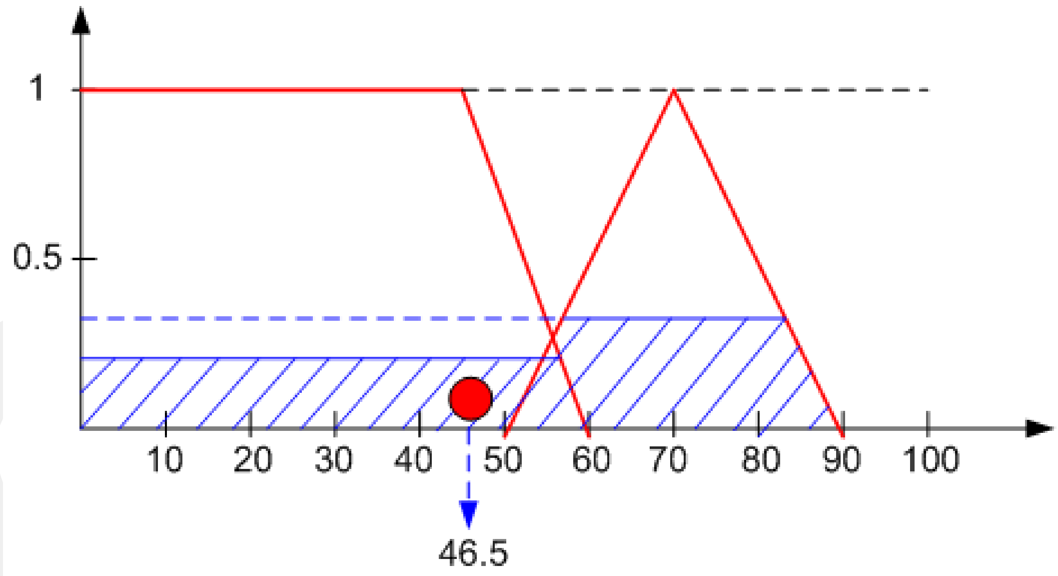


Figure 16: Calculated Centre of Gravity

Same problem is transferred to MATLAB Fuzzy Logic Toolbox. For P = 86 and E= 82, the result shown in **Figure 17**, which is the same result with the one obtained above.

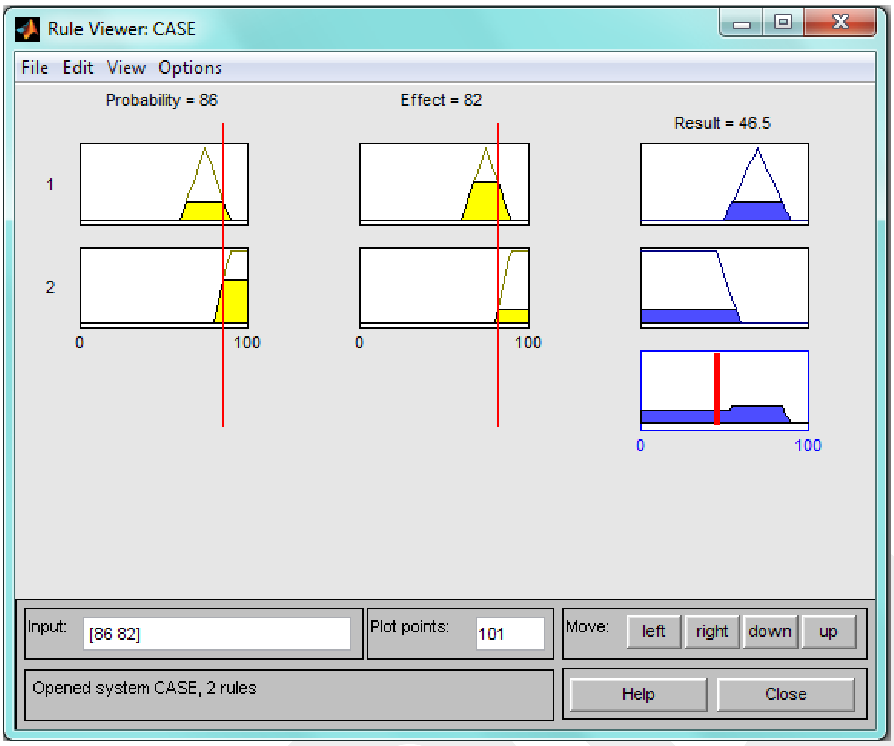


Figure 17: MATLAB Fuzzy Logic Toolbox Result

It is a well-known situation that fuzzy logic contains subjectivity characterization differs from expert to expert. Accordingly the critical point is the categorization of the events. It seems very difficult for the experts to give the exact inputs and outputs, several parameters are effective on the accuracy of the experts such as duration of experience, profession, content of the past experience etc. So, for the fuzzy applications, it should be considered that how expert are the experts. For the application of the proposed model, experts with sufficient experience and having different point of views by covering all the possible domains should be selected.

For the case given above, if the sensitivity of the expert opinions is considered, following error cases are tried with the same model and the change on the result is observed. Observations are given in **Table 16** and **Table 17**, for probability and severity respectively.

Table 16: Sensitivity of the Probability of Occurrence

Case #	Probability of Occurrence (0-100)	Severity (0-100)	Result	Result Category
1.	75	82	70	Moderate
2.	76	82	70	Moderate
3.	77	82	70	Moderate
4.	78	82	70	Moderate
5.	79	82	70	Moderate
6.	80	82	70	Moderate
7.	81	82	58.9	Failure & Moderate
8.	82	82	52.2	Failure & Moderate
9.	83	82	51.3	Failure & Moderate
10.	84	82	50.1	Failure & Moderate
11.	85	82	48.5	Failure
12.	86	82	46.5*	Failure
13.	87	82	43.8	Failure
14.	88	82	40.2	Failure
15.	89	82	35.5	Failure
16.	90	82	29	Failure
17.	91	82	29	Failure

18.	92	82	29	Failure
19.	93	82	29	Failure
20.	94	82	29	Failure
21.	95	82	29	Failure

Table 17: Sensitivity of the Severity

Case #	Probability of Occurrence (0-100)	Severity (0-100)	Result	Result Category
1.	86	70	70	Moderate
2.	86	71	70	Moderate
3.	86	72	70	Moderate
4.	86	73	70	Moderate
5.	86	74	70	Moderate
6.	86	75	70	Moderate
7.	86	76	70	Moderate
8.	86	77	70	Moderate
9.	86	78	70	Moderate
10.	86	79	70	Moderate
11.	86	80	70	Moderate
12.	86	81	54	Failure & Moderate
13.	86	82	46.5*	Failure
14.	86	83	42.2	Failure

15.	86	84	39.5	Failure
16.	86	85	37.6	Failure
17.	86	86	36.1	Failure
18.	86	87	34.4	Failure
19.	86	88	32.4	Failure
20.	86	89	30.2	Failure
21.	86	90	27.6	Failure
22.	86	91	27.6	Failure
23.	86	92	27.6	Failure
24.	86	93	27.6	Failure
25.	86	94	27.6	Failure
26.	86	95	27.6	Failure

The change on the result for probability and the effect can be seen in **Figure 18** and **Figure 19** respectively.

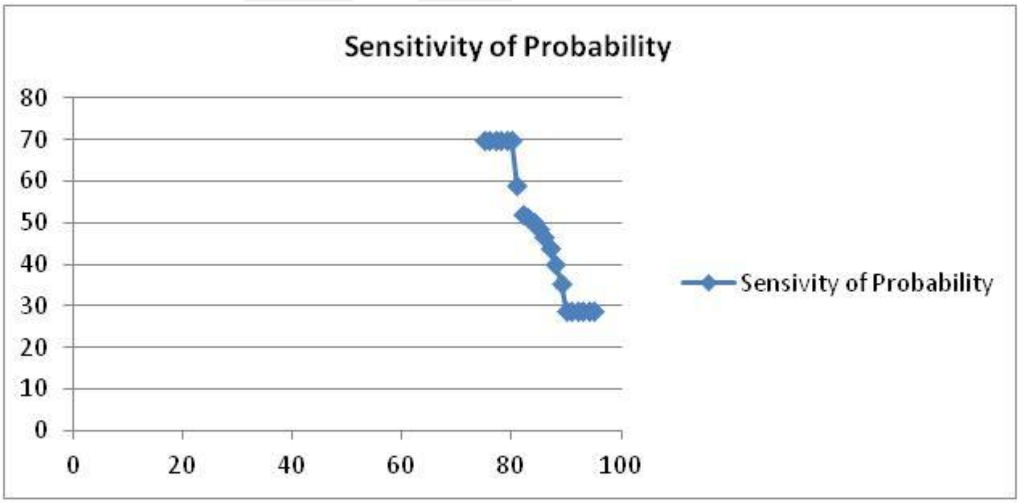


Figure 18: Change of the Result for Probability

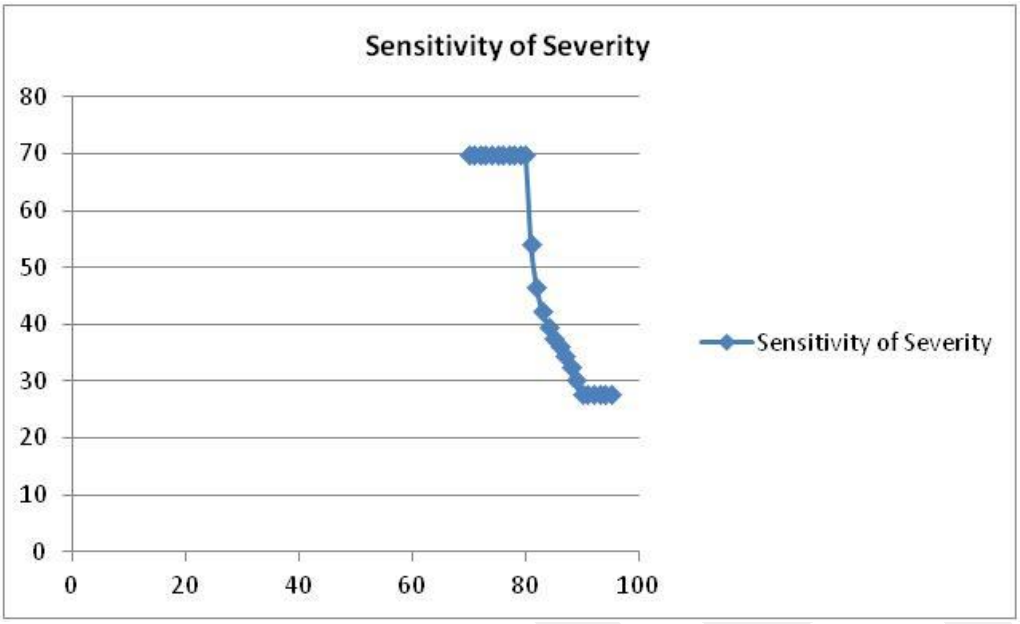


Figure 19: Change of the Result for Severity

As it can be seen from **Table 16** and **Table 17**, for correct output category, it is required to get correct regions in terms of input membership functions, if the output evaluation criteria are same with the output membership function categories. This situation also emphasizes the importance of expert opinions. Deviation of approximately 10% of expert opinion results with a deviation between ~30 to 70, at a scale of 100.

CHAPTER 5

APPLICATIONS

The fuzzy method is applied on mechatronic systems such as a typical Mobile Robot System (MRS) and a missile system. Since MRS is a mechatronic system such as missiles, the method proposed in this thesis can also be applied on MRS to estimate system reliability.

5.1. Mobile Robot System

A Mobile Robot System (MRS) is a simple mechatronic system which is designed for following a line. MRS is composed of the components given in **Figure 20**.

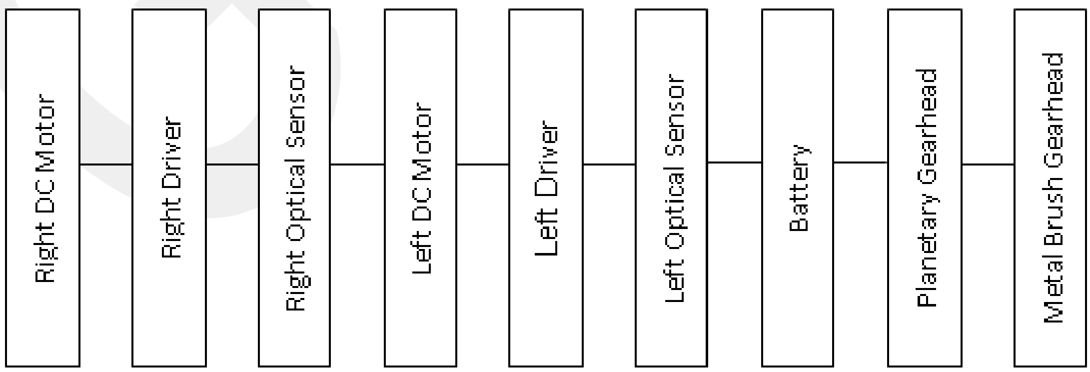


Figure 20: RBD of MRS

The operation of MRS can be summarized as given in **Figure 21**.

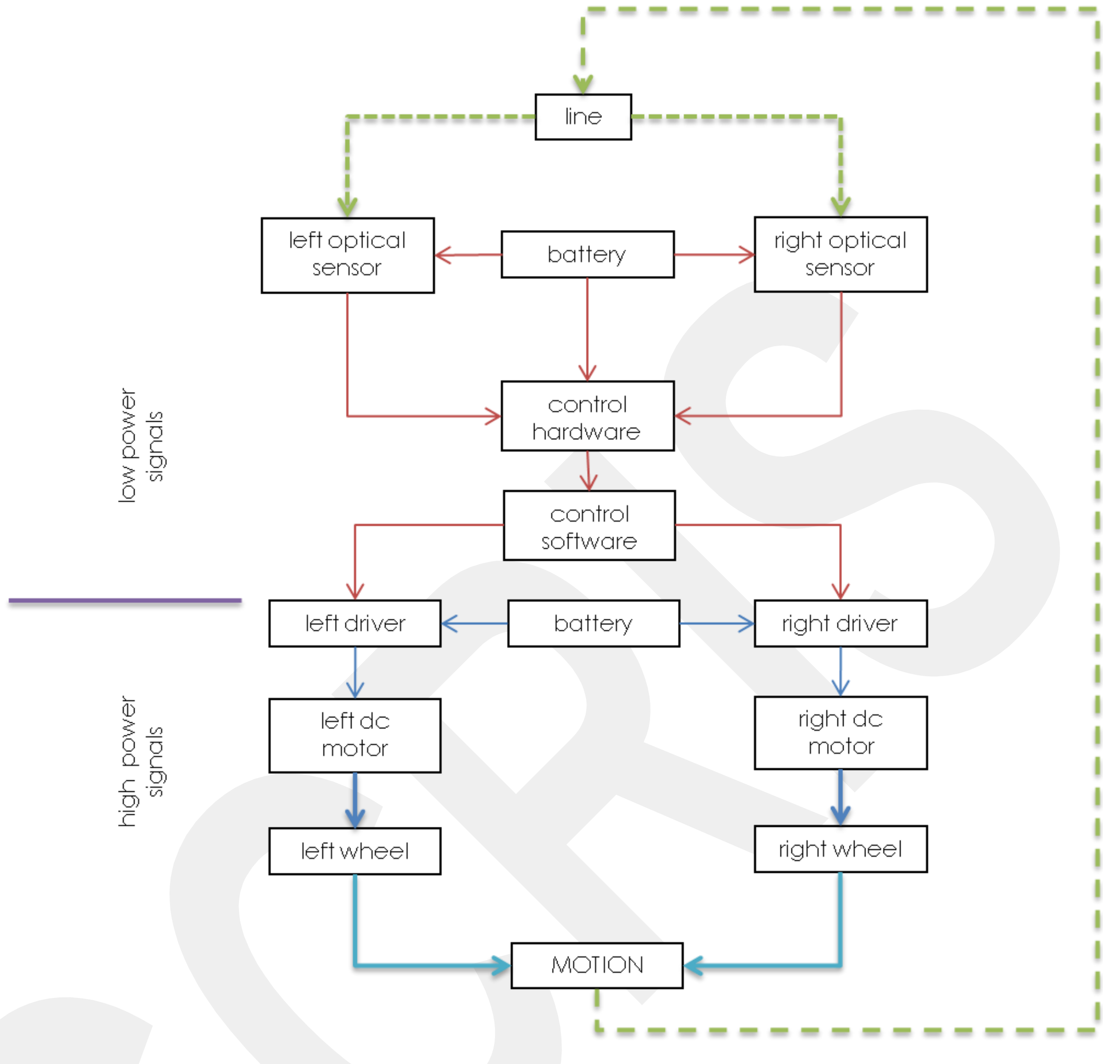


Figure 21: Operational Flow of a Typical MRS

Typical failures of an MBS can be categorized as follows;

- Failure of optical sensor(s)
- Failure in the interface circuitry
- Failure in the motor drivers
- Failure in the software
- Failure in battery pack

FMEA study is performed and failure modes for a typical MBS are defined. Experts on MBS design are asked for the probability of occurrence, severity category and effect on system reliability for each failure mode according to the classifications in **Table 13** and **Table 14**. Also the number of the rules can be extended for each failure with different probability, effect and result. The averages values of expert opinions for the failure modes of MRS are given in **Table 18**.

Table 18: Failures of MRS

Failure	Cause of the Failure	Category of the Probability of Occurrence	Category of the Severity	Result
MRS is not moving	Battery is flat	E	5	3
	Battery is faulty	E	2	1
	Power cables of Left Driver and Right Driver are damaged	E	1	1
	Power cables between Left Driver and Left DC motor and also Right Driver and Right DC motor are damaged	E	1	1
	Left DC Motor and Right DC Motor are failed	E	1	1
	Left DC Motor does not transmit rotation to Left Wheel (problems with transmission interface)	D	2	1
	Right DC Motor does not transmit rotation to Right Wheel (problems with transmission interface)	D	2	1
	Control software do not generate motion signal	E	1	1

Failure	Cause of the Failure	Category of the Probability of Occurrence	Category of the Severity	Result
	The motion signal generated by Control Software can't be received by Left Driver and Right Driver	E	2	1
	Power cable of Control Hardware is damaged	E	1	1
MRS is not following the line	Control software generates faulty motion signal	D	1	1
	One of the optical Sensors is failed	D	2	1
	Transmission delay of Control software to send motion signal to Left Driver on time	E	3	1
	Transmission delay of Control software to send motion signal to Right Driver on time	E	3	1
MRS can't perform turning to the left	Left Optical Sensor is failed	E	2	1
	Power cable of the Left Optical Sensor is damaged	E	1	1
	Signal from Left Optical Sensor to Control Hardware can't be transmitted	E	1	1
MRS can't perform turning to the right	Right Optical Sensor is failed	E	2	1
	Power cable of the Right Optical Sensor is damaged	E	1	1

Failure	Cause of the Failure	Category of the Probability of Occurrence	Category of the Severity	Result
	Signal from Right Optical Sensor to Control Hardware can't be transmitted	E	1	1
MRS is not speeding at desired level	Low voltage level of Battery	B	4	3
	Left Driver and Right Driver do not provide desired rotation to motors	E	1	3

Figure 22 summarizes the approach of constructing the membership functions and their interactions using *linguistic* rules used in the fuzzy system.

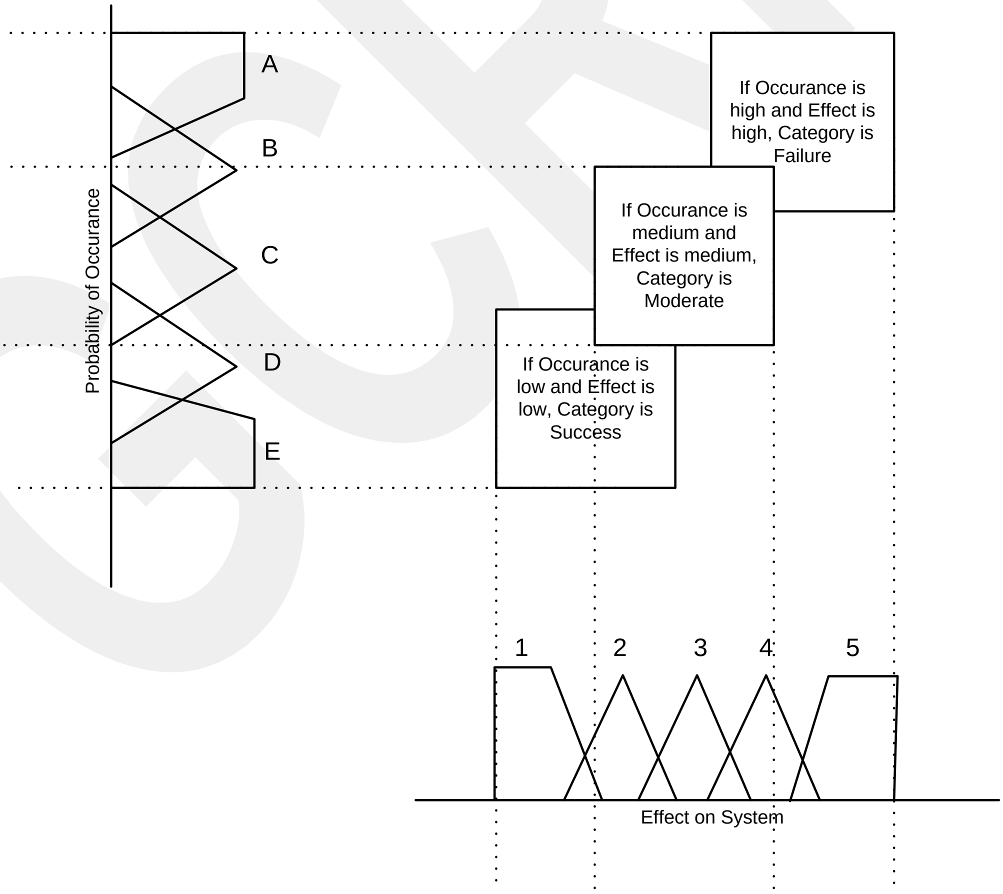


Figure 22: Membership Functions

Fuzzy system for MRS is modeled in MATLAB 7.1 Fuzzy Logic Tool [22]. Fuzzy rules obtained by the FMEA study with extensions are entered on Rule Editor as given in Figure 23.

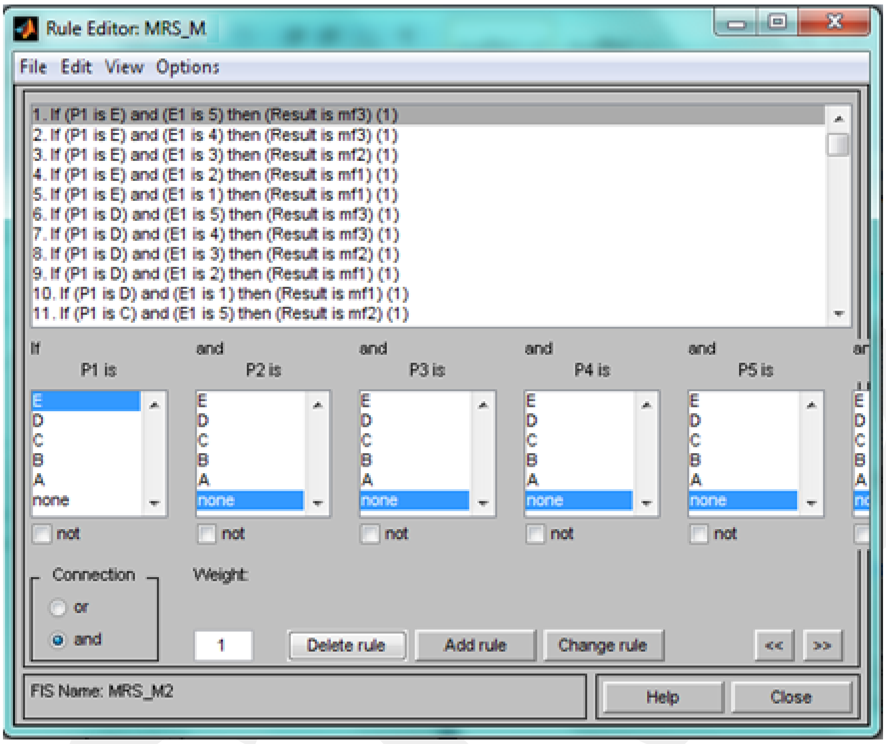


Figure 23: Fuzzy Rules for MRS

Membership functions are constructed as explained in Section 4.4.1, according to **Figure 22**, **Table 13** and **Table 14**.

The Mamdani-type fuzzy inference is obtained as given in **Figure 24**. Also failure modes and the reliability can be analyzed via surface viewer for each pair of the events. A sample is given in **Figure 25** for representation of the surface for Probability-1 and Effect-1. According to **Figure 25**, it can be seen that for the decreasing values of the Probability-1 (from improbable to frequently according to **Table 13**) and decreasing values of Effect-1 (from catastrophic to negligible according to **Table 13**), the system reliability (O1) also increases as expected.

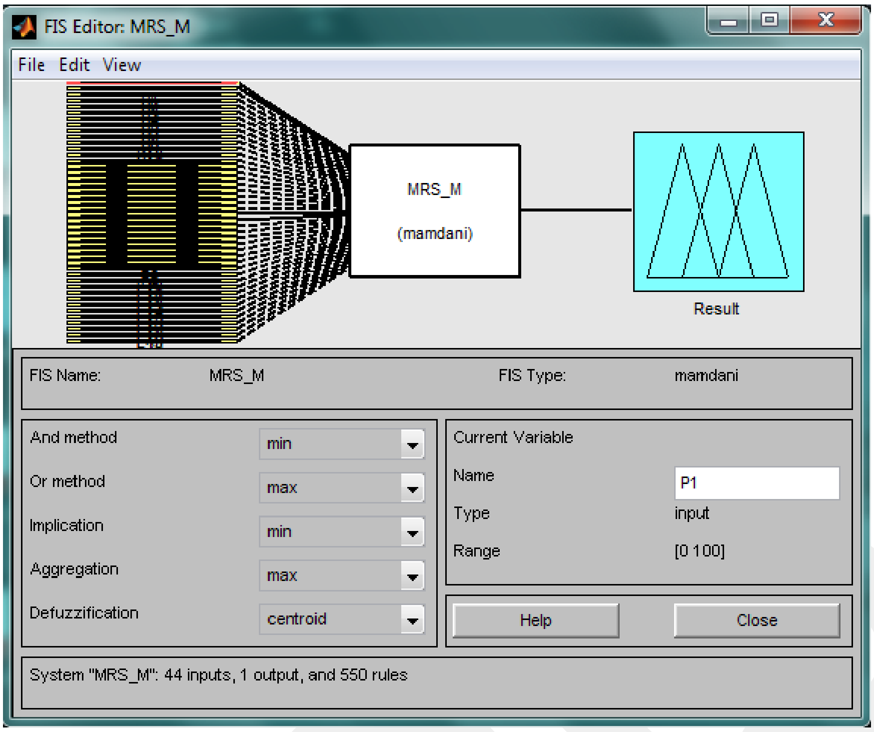


Figure 24: Fuzzy Inference for MRS

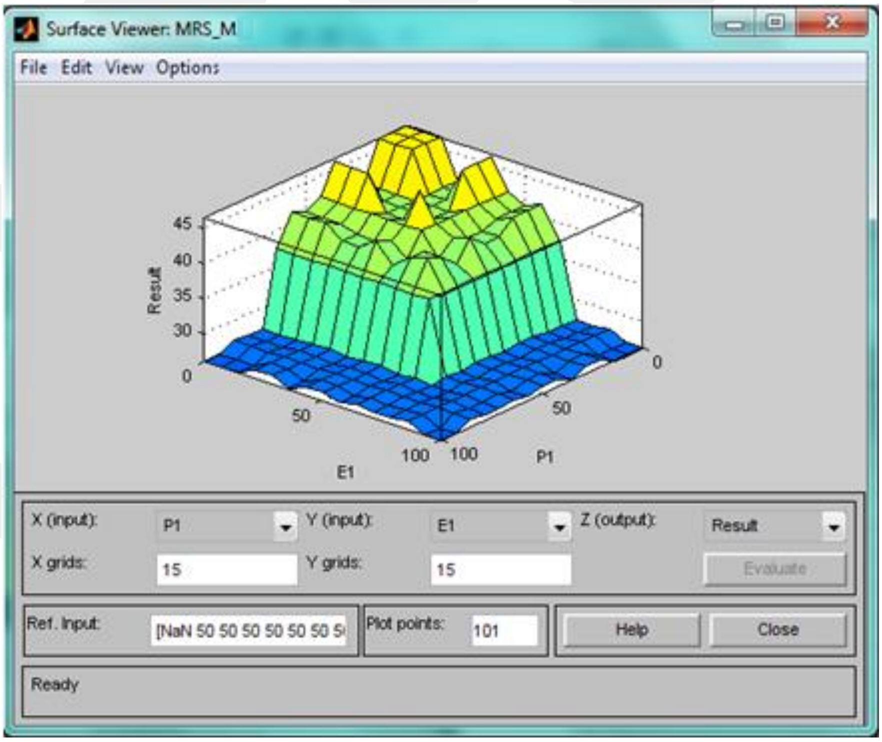


Figure 25: Surface for E1 and P1 for MRS

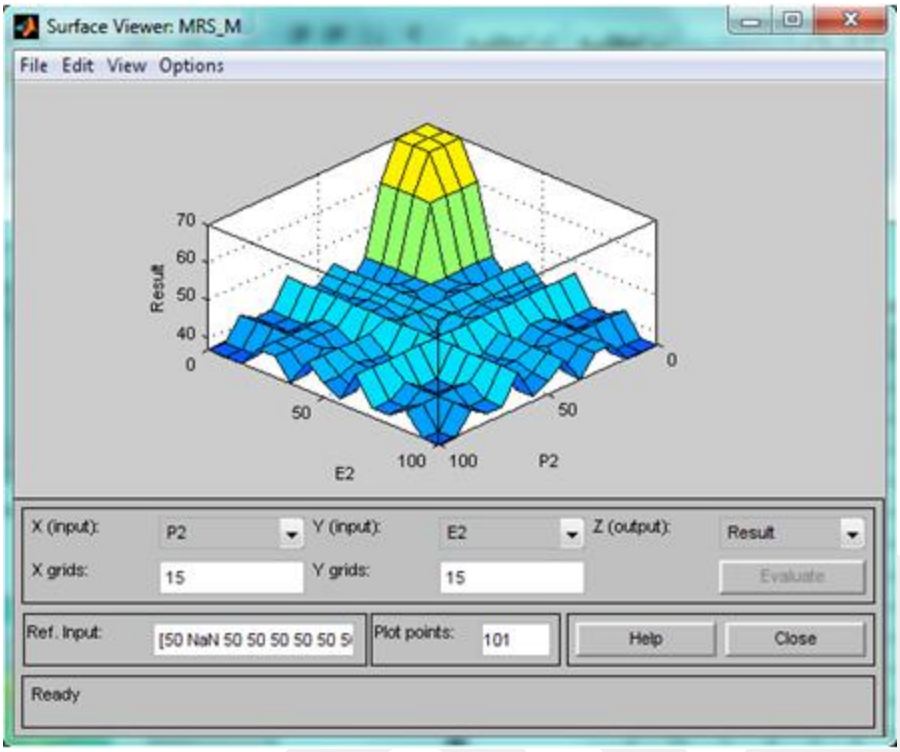


Figure 26: Surface for E2 and P2 for MRS

For the different values of the input parameters, deviation on the system reliability can be obtained easily. As a defuzzification method, Centroid is used in the model. Results are obtained as given in **Table 19** including 3 sample cases for evaluation.

Table 19: Results

Event#	Prob.	Eff.	Prob.	Eff.	Prob.	Eff.
1	0	0	50	50	0	0
2	0	0	0	0	50	50
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0
7	0	0	0	0	0	0

8	0	0	0	0	0	0
9	0	0	0	0	0	0
10	0	0	0	0	0	0
11	0	0	0	0	0	0
12	0	0	0	0	0	0
13	0	0	0	0	0	0
14	0	0	0	0	0	0
15	0	0	0	0	0	0
16	0	0	0	0	0	0
17	0	0	0	0	0	0
18	0	0	0	0	0	0
19	0	0	0	0	0	0
20	0	0	0	0	0	0
21	0	0	0	0	0	0
22	0	0	0	0	0	0
Centroid	92.5		80.3		46.6	

According to **Table 19**, for event#1 and event#2, same values for probability and effect are entered. However the results obtained are significantly different since the marginal effects of the events are different. For the system when only event#1 is available with indicated inputs, reliability can be evaluated as 80.3 at a scale of 92.5. And when only event#2 is available with indicated inputs, reliability can be evaluated as 46.6 at a scale of 92.5.

This means that event#2 has bigger marginal effect on system reliability, and priority to take required precaution for that event should be given when compared with event#1. As it can be understood from this example, all the event pairs can be evaluated in system design. This feature of the model provides clues to select the most critical events in terms of improving system reliability in design phase since there is not any tool providing this opportunity.

5.2. Hypothetical Missile System

For assessing the reliability of a missile, it is not adequate to evaluate a single value of reliability, since different phases have different reliability values. Classical approaches are modeling the system with all the components in Bill of Material (BOM), finding the reliabilities of them in given reliability software libraries, modeling the system, and as a result getting the reliability of the whole system.

However every component in BOM does not have direct contribution on system reliability. Some of the components are related with phases such as storage reliability, some of them are related with guidance reliability and so on. Accordingly, components have effects on phase reliabilities if they are actively used in that phase, and also have effect on overall reliability in coordination of the weight of the related phase.

To overcome this insufficiency in this thesis, missile reliability is defined as the probability of hitting the target as intended under the correct firing conditions. Accordingly, missile reliability can be expressed as a relation of different phases such as pre-transportation, transportation, launch and flight. According to this definition, reliability of the missile should be categorized in the phases as defined in **Figure 27**.

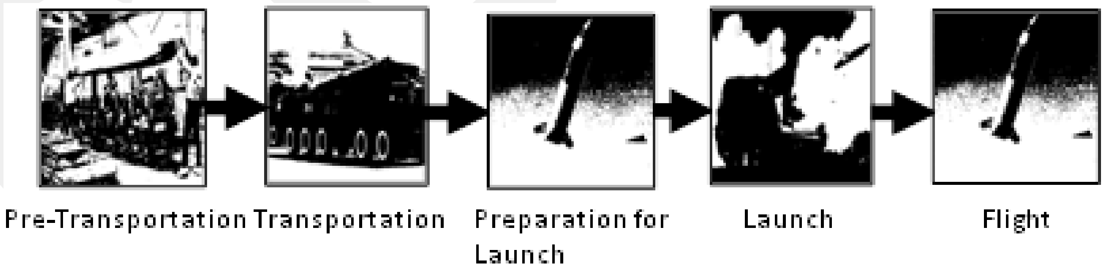


Figure 27: Missile Operational Phases

Each phase's reliability is estimated with different combinations of the components in BOM since all the components do not affect every phase of the operation of a missile.

And also every component in these phases have different amount of contribution to the phase reliability. For example, the reliability of the seeker and the frame which holds the cables in the missile are not at equivalent significance levels in terms of Mission Reliability.

For this purpose, FMEA is used to list the possible events that a missile can face at different operational phases, categorize them according to their probabilities of occurrence and detecting their effects on the system.

Expert opinion may be available from several experts, each of whom may provide information regarding the reliability of different subsets of components, and the quality of information obtained from each may vary. Efficiently incorporating expert knowledge into the estimation method of system reliability can therefore be a complicated task.

For evaluating the expert opinions and combining them with FMEA study in an effective way, the proposed method is based on Fuzzy Logic which is successful in combining complex data with a detailed rule base system occurring from FMEA in this thesis.

In this thesis, the probabilities of occurrence and their effects on system are provided from two experts in each of the related field of the system. If more experts are available, it's better to get a large number of opinions from several experts. Average values of the expert opinions are taken to represent the probability and the effect parameters. A fuzzy model is developed to combine these expert opinions with the rules defined in FMEA for getting the reliability estimations of each phase.

Fuzzy System for a typical missile system is modeled in MATLAB 7.1 Fuzzy Logic Tool [22]. Fuzzy rules obtained by the FMEA study is entered on Rule Editor as given in **Figure 28**.

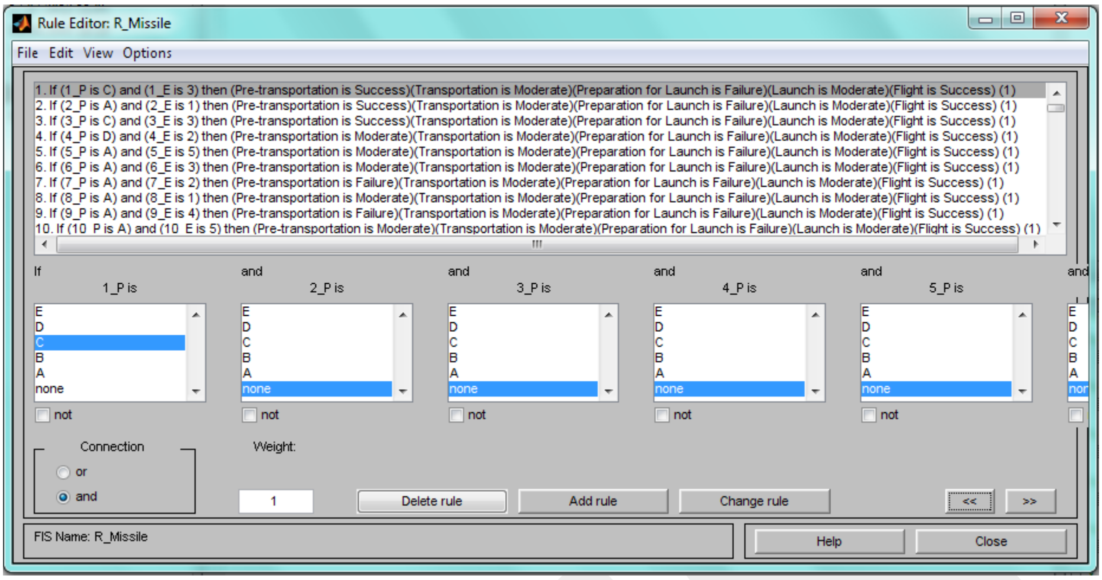


Figure 28: Fuzzy Rules for Typical Missile System

Input and output membership functions are constructed as explained in **Figure 22**, **Table 13** and **Table 14**. Since the output situations of the missile reliability can be evaluated as success, moderate and failure, which can't be separated certainly, Mamdani type fuzzy inference is used as given in **Figure 29**.

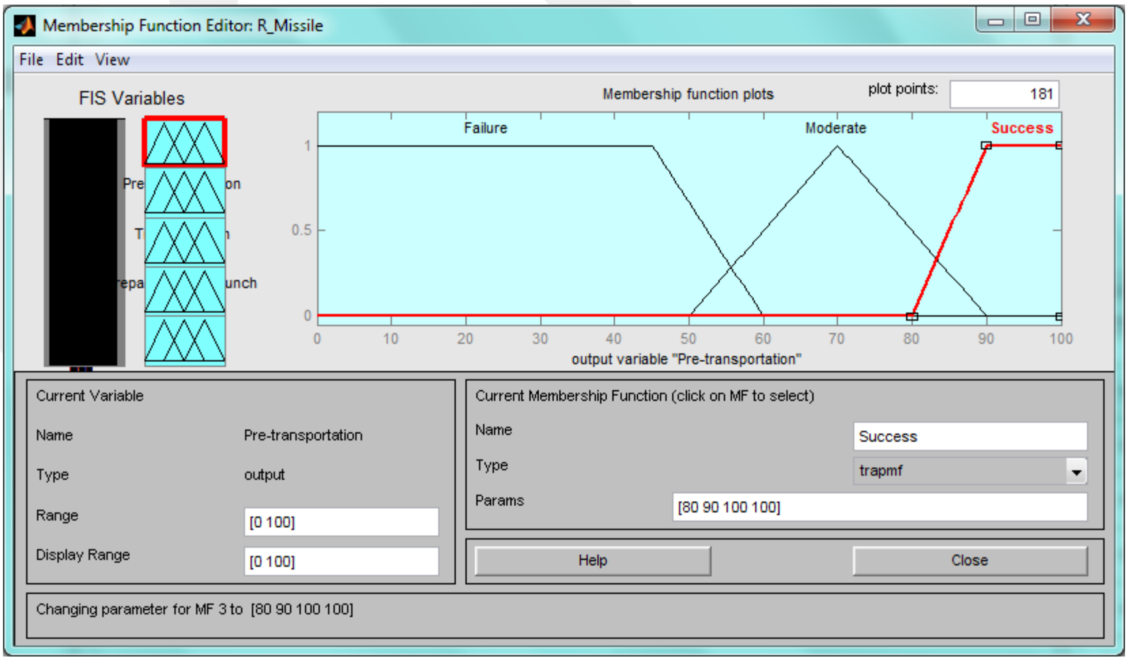


Figure 29: An Output Membership Function for Typical Missile System

The Mamdani type fuzzy inference is obtained as given in **Figure 30**. This model provides the opportunity to evaluate the effects of any change with an event on all 5 phase reliabilities at the same time. Also events and the reliability can be analyzed via surface viewer for each pair of the events.

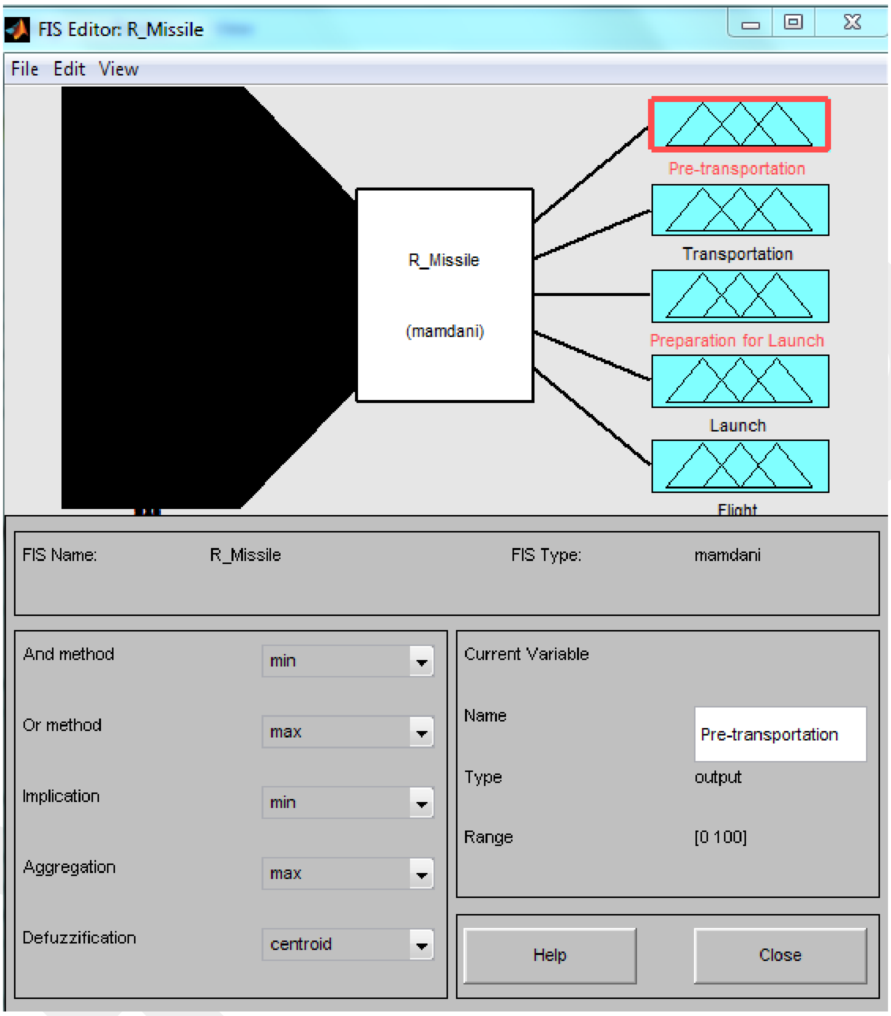


Figure 30: Mamdani Type Fuzzy Inference for MRS

For each operational phase, different behaviors of the reliability during design phase are handled in a model as mentioned in **Figure 30**. Also this approach can be applied for a wide variety of systems under development.

CHAPTER 6

DISCUSSIONS AND CONCLUSION

In this thesis, FMEA technique and expert opinions are combined via a fuzzy logic system and this system is used to estimate the reliability of a missile system during design phase. One of the contributions obtained with this thesis is providing a decision support system at design phase for missile reliability estimations where there is not any alternative tool. For this estimation, all the operational phases such as storage, transportation, preparation for launch, launch and flight of a missile are taken into account. If the probability or the effect of any event in the FMEA is changed, its effects on different operational phases can be obtained easily via the developed fuzzy model. Also instead of modeling RBD of all the components/items in BOM of a missile which can be called as down-to-top approach, all the possible events in a life cycle of a missile are taken into consideration and used in a fuzzy logic system with FMEA, which can be called as top-to-down approach. The advantage of this approach is eliminating the unrelated events or components from the phase reliabilities.

The model proposed in this thesis can be applied to a wide variety of systems under development, not only to missile systems. FMEA and expert opinions are sufficient to construct the fuzzy model of the system under development.

The proposed model is also tried on a typical MRS, which is a simple and well-known mechatronic system studied in term projects, competitions etc. It is also easy to find experts for this case and this makes the case ideal for application of the proposed model. However it should be noted that the performance of the proposed model is strongly dependent on the quality of the expert opinions and FMEA. The purpose of the proposed model is to provide a decision support system during design phase and to get a representative reliability estimation, not to find the final reliability value of the system.

Typical missile system reliability is also modeled with the proposed fuzzy logic system with five operational phases, namely pre-transportation, transportation, preparation for launch, launch and flight. It is possible to see the effect of any change of any failure mode on all the operational phases of the missile system. This feature of the model provides clues to select the most critical events in terms of improving system reliability in design phase since there is not any tool providing this opportunity. With this opportunity the proposed model serves as a decision support system for reliability estimation for missile systems under development, accordingly design optimization for reliability can be performed by taking required precautions on specific failure modes.

The reliabilities obtained via the proposed model is compared with the results obtained via reliability prediction software; however some of the components did not match with the ones in the reliability library. It is obviously not possible to find all the components used in all the industries in reliability libraries. For the components that could not be matched in the library, characteristics of the components are entered. The reliabilities obtained by reliability prediction software and the proposed model can be considered as close enough values for design phase.

As stated before, the quality of the result found via the proposed model is strongly dependent on the events defined in FMEA and expert opinions. The FMEA should include all the possible events that the system can be exposed to during its lifecycle. However, the data obtained later can also be handled in the proposed model by defining the new failure mode or updating the existing failure modes in the fuzzy system via making some minor code edits. Also the obtained results should be considered as early reliability estimations which provide a judgment tool for decisions at design phase.

For further studies in this subject, applications on different reliability analysis techniques, especially Fault Tree Analysis (FTA) can also be integrated with fuzzy logic system. Also integration of the known MTBF values to the proposed model for the previously used or qualified components in BOM can be studied in detail.

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